Lecture 6

Red-Black Trees: Insertion, Deletion

Source: Introduction to Algorithms, CLRS

RB-Trees: Insertion Cases

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Gets converted to Case 3 (Case 2)

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Gets converted to Case 3

Fix-up will be enough to terminate the process

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- If parent of z does not exist, make z **black** and **exit**.

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We will see the fix ups assuming parent of z is a left child.

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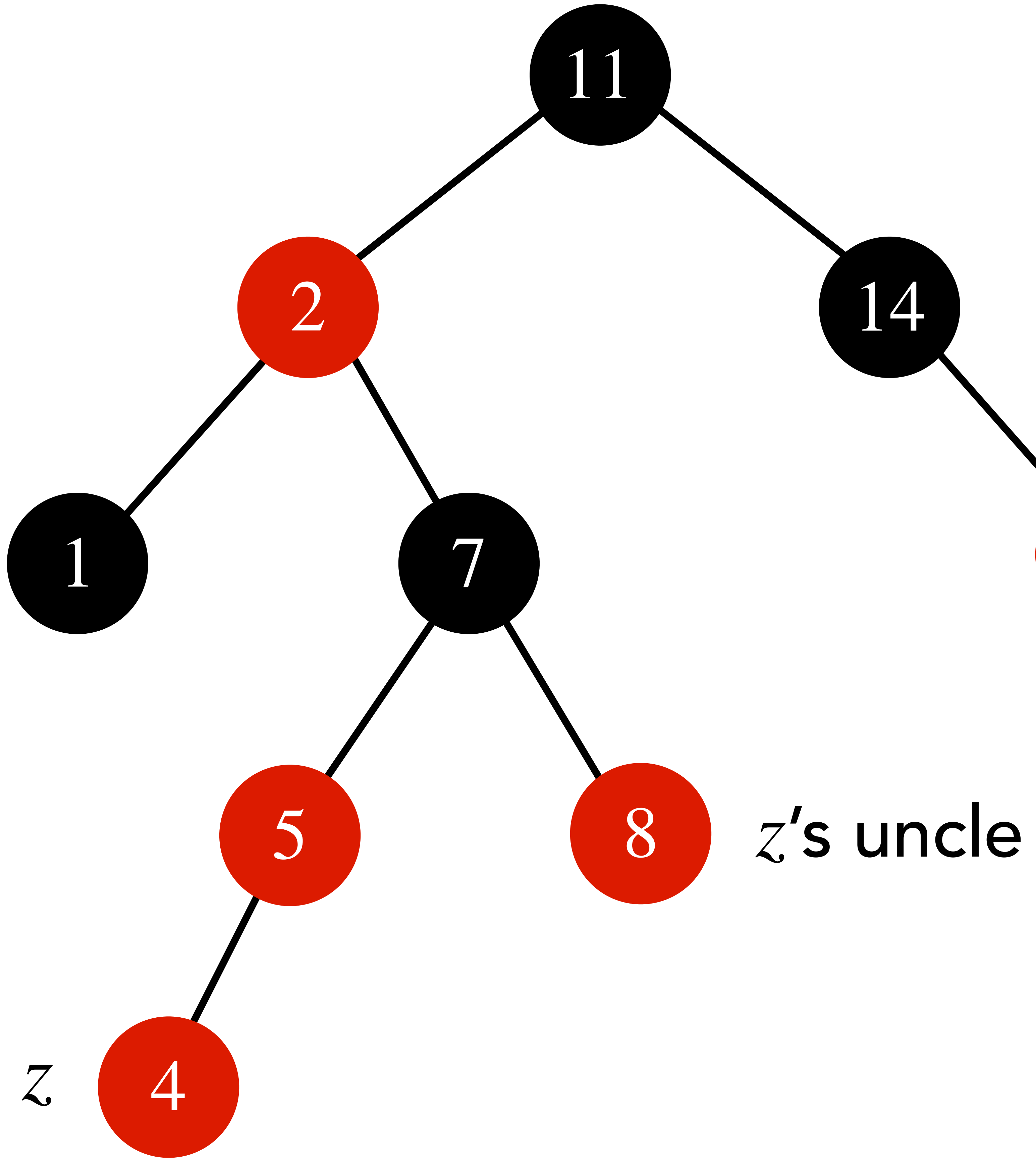

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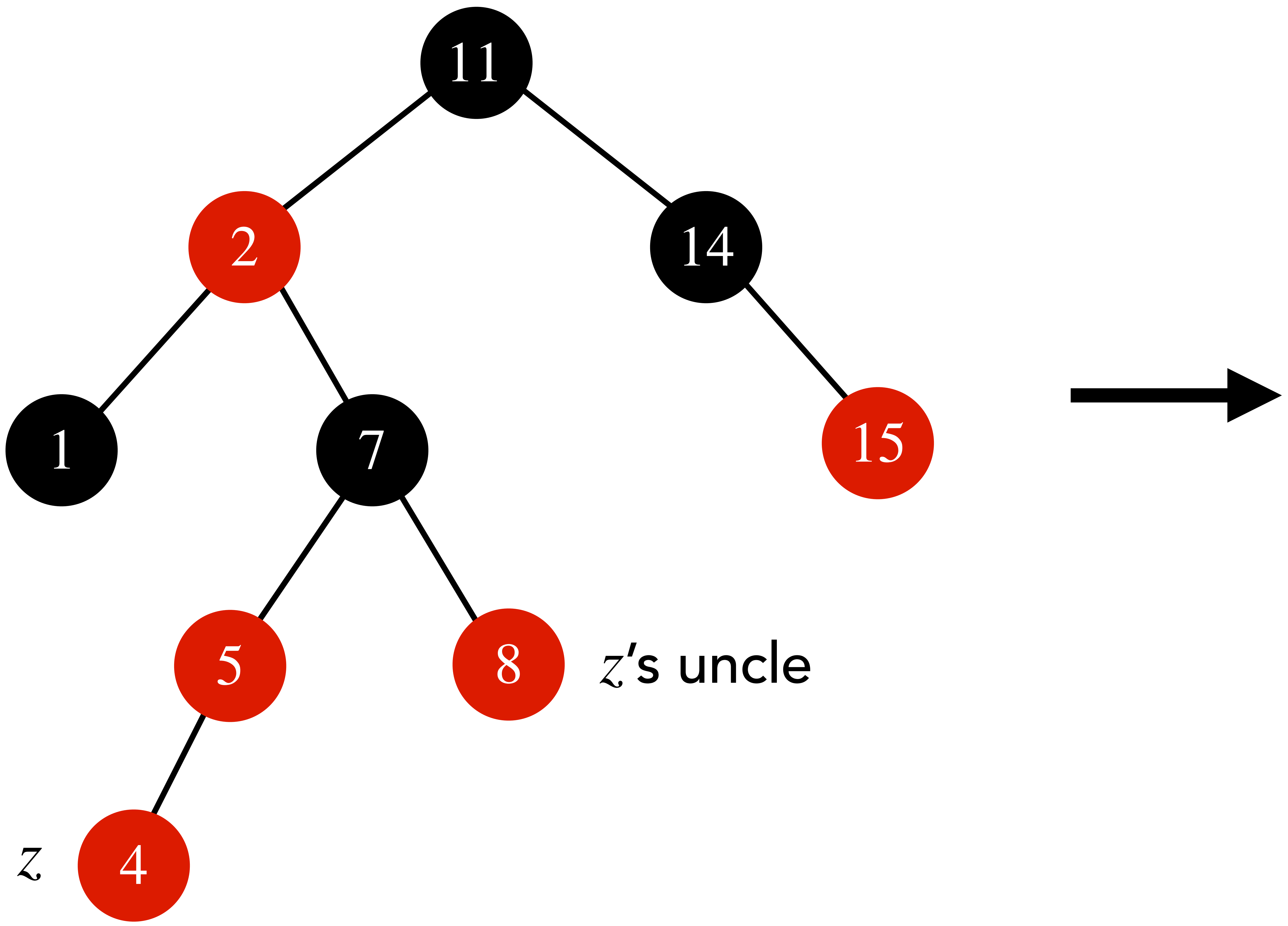

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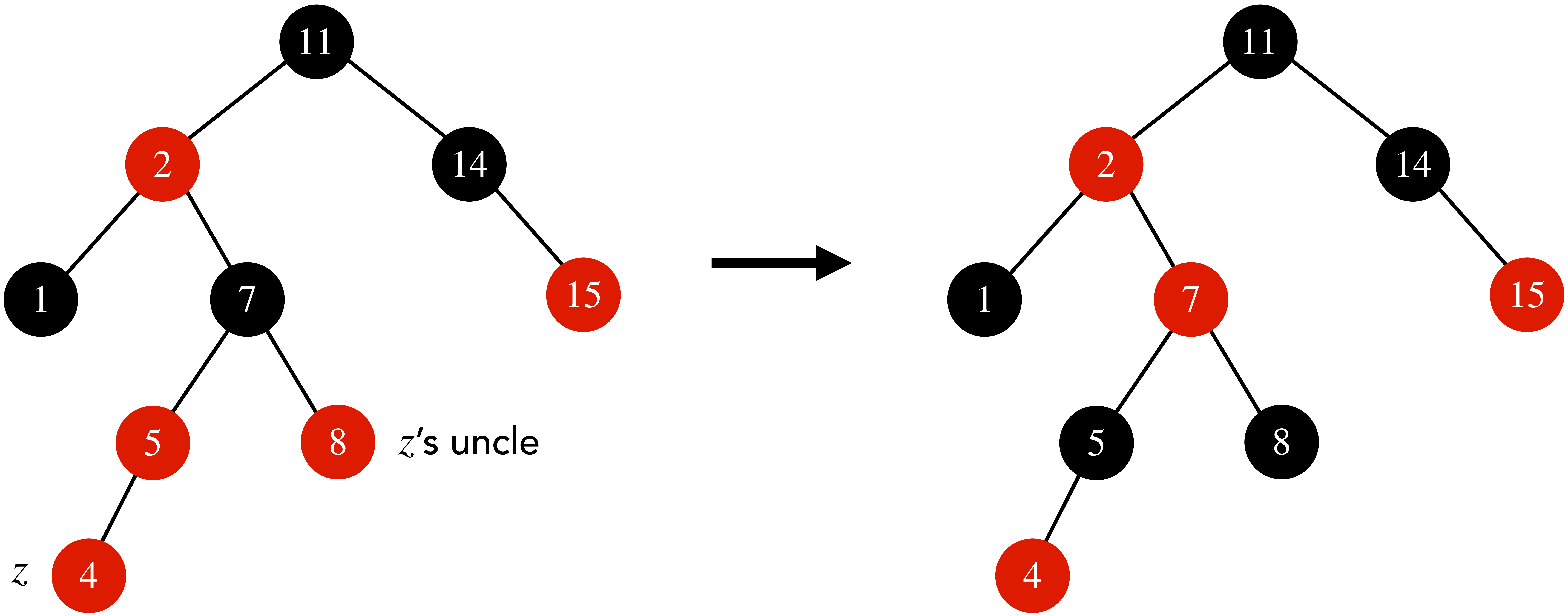

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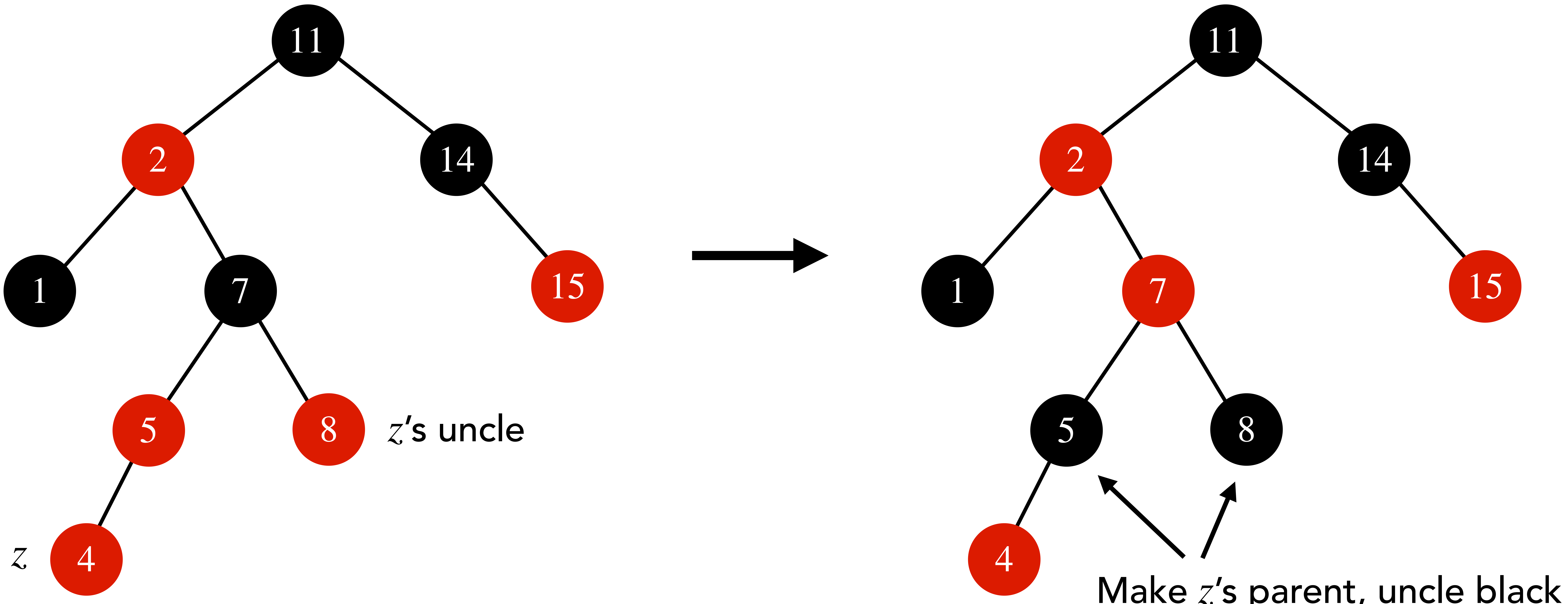

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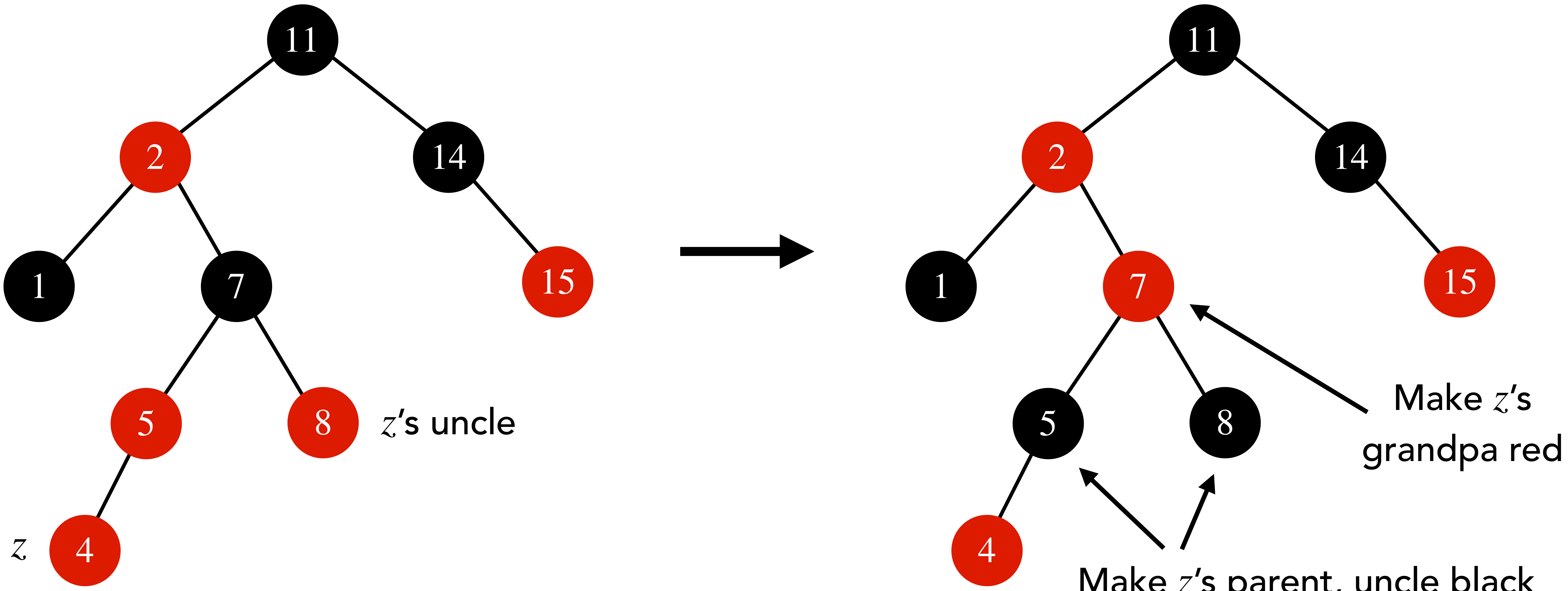

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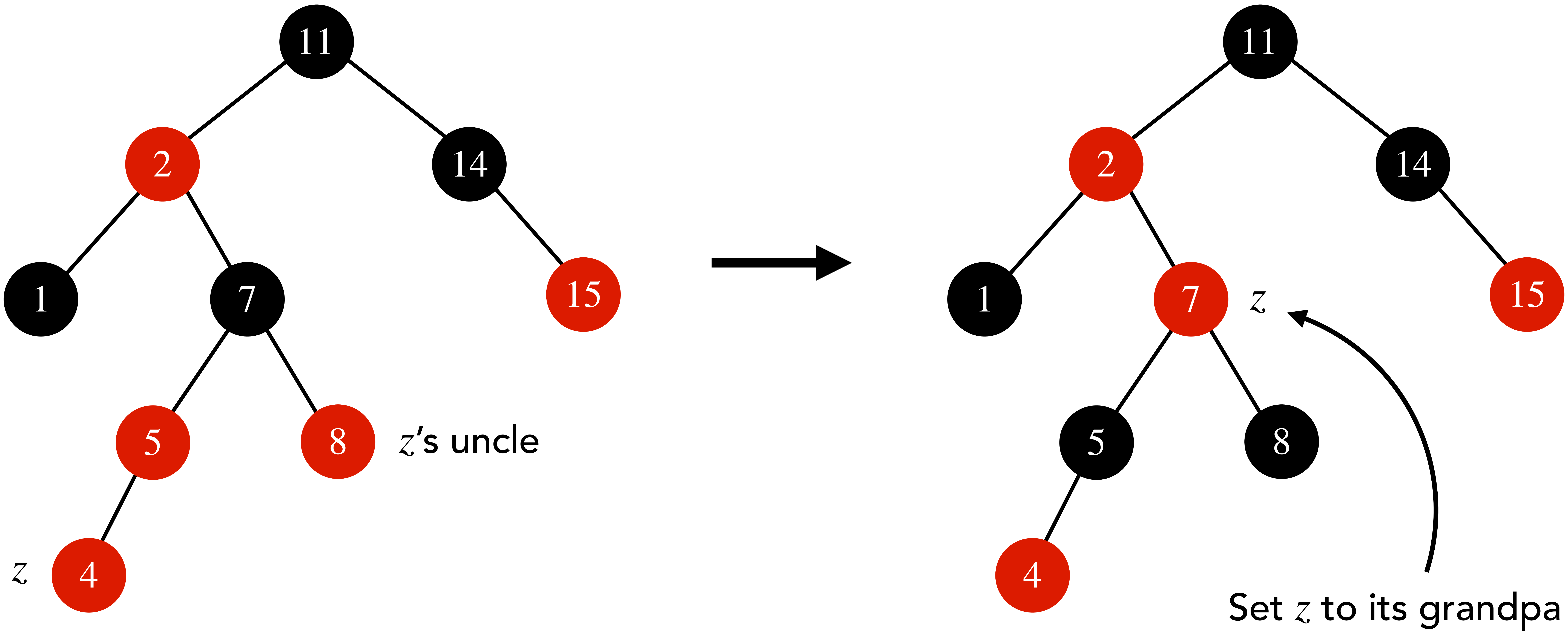
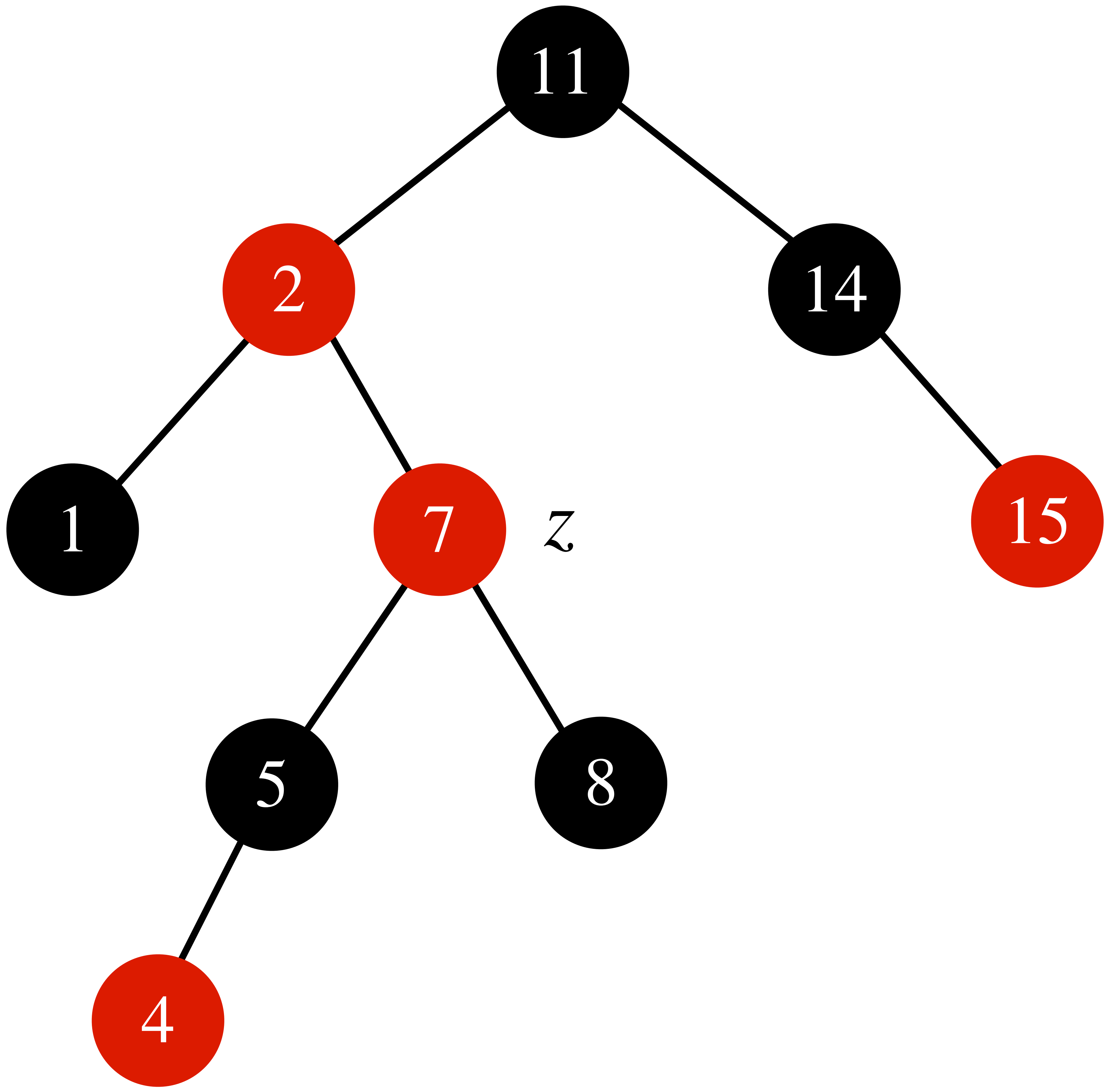


RB-Trees: Insertion Case 2

Case 2: z's uncle is black and z is a right child.



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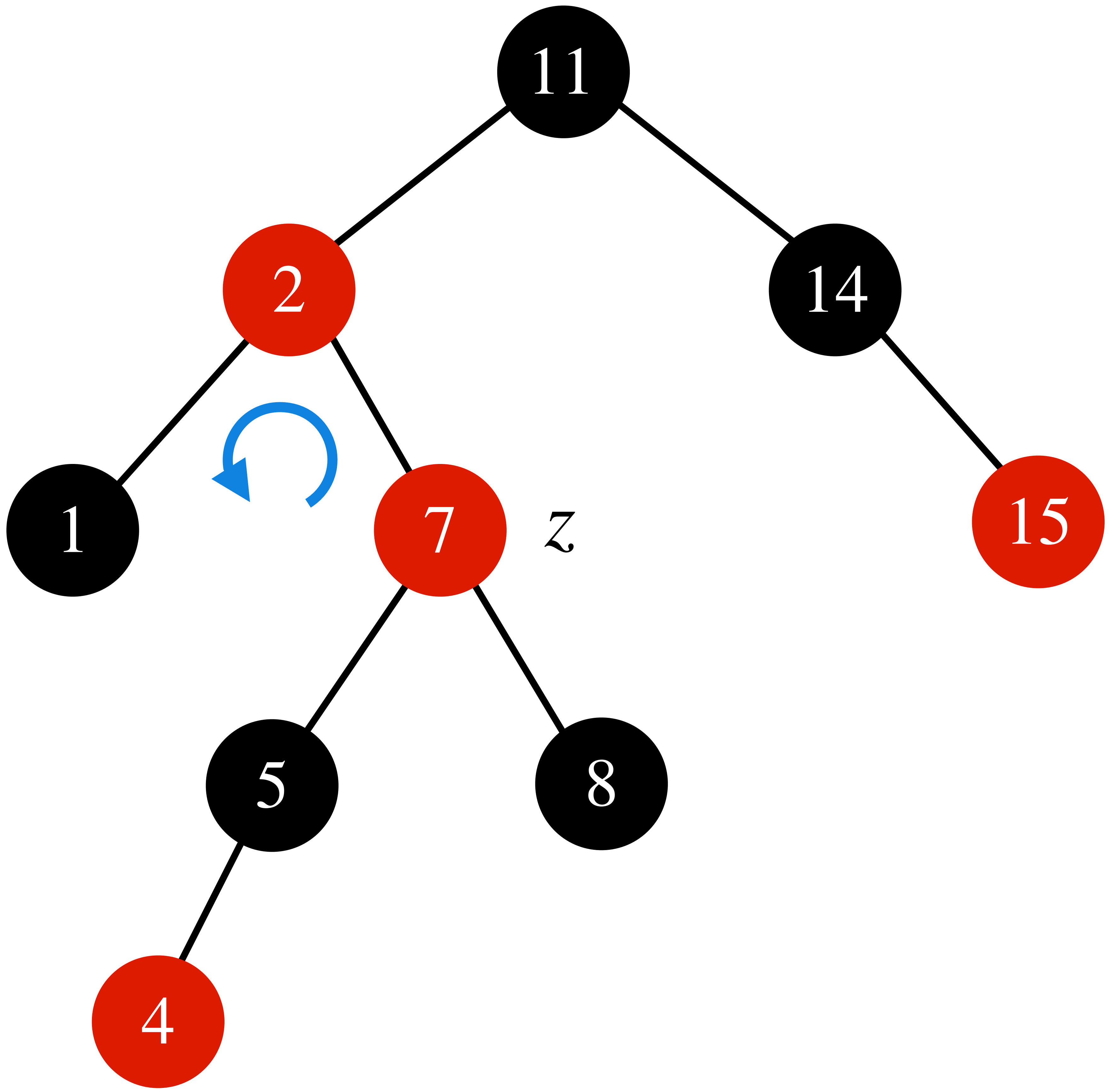

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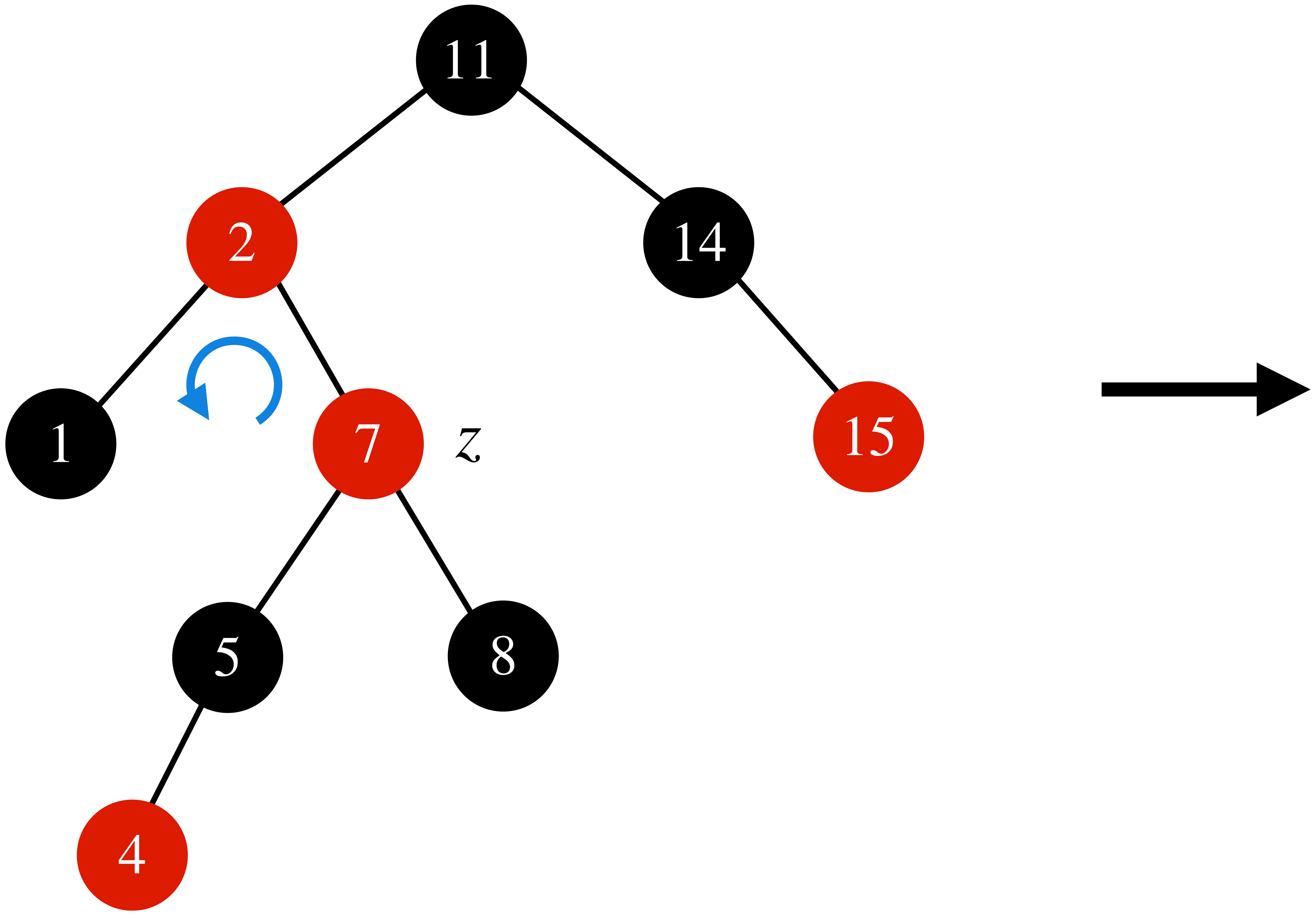

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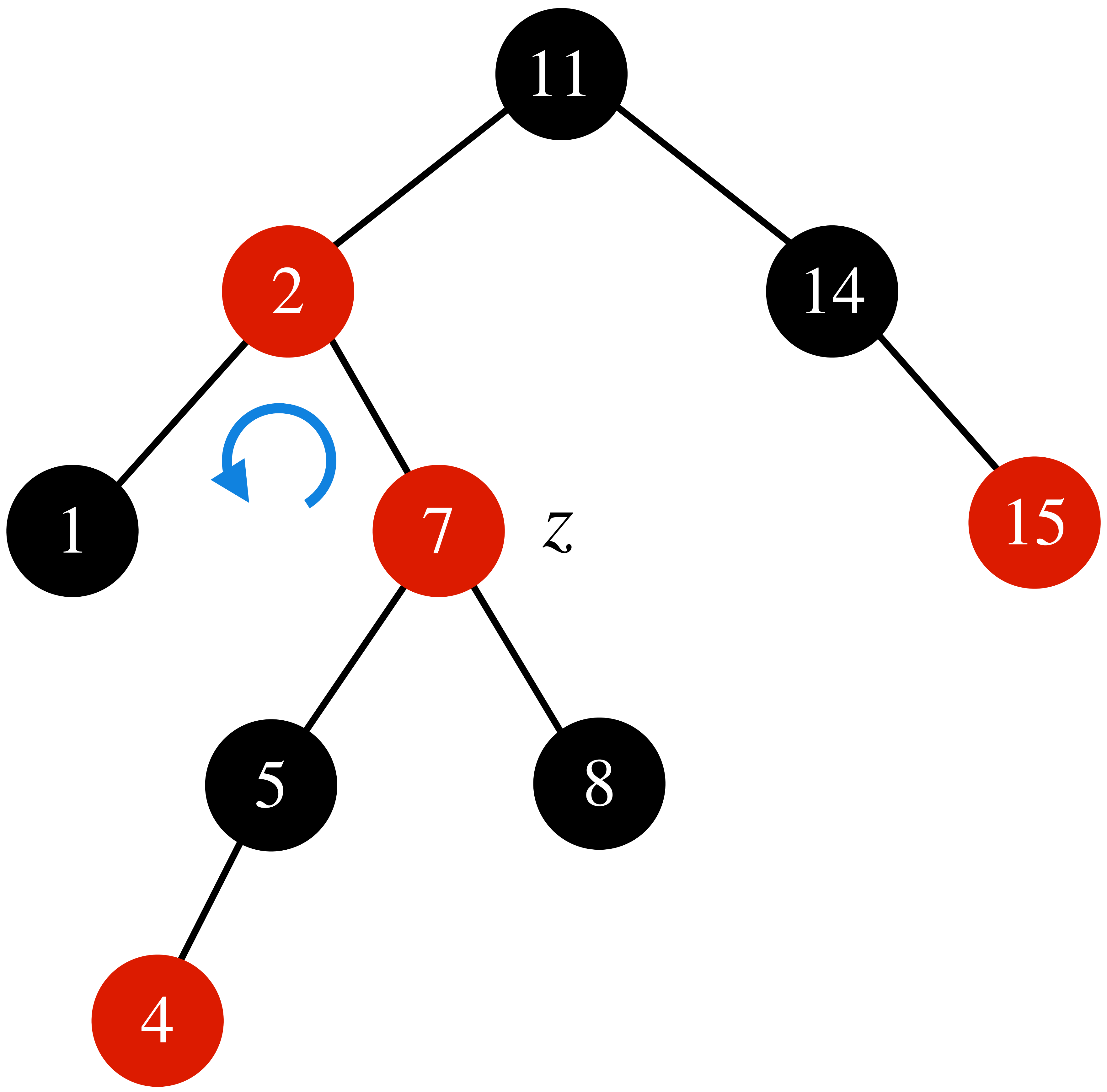

Set z to its parent and Left-rotate(T, z)

RB-Trees: Insertion Case 2

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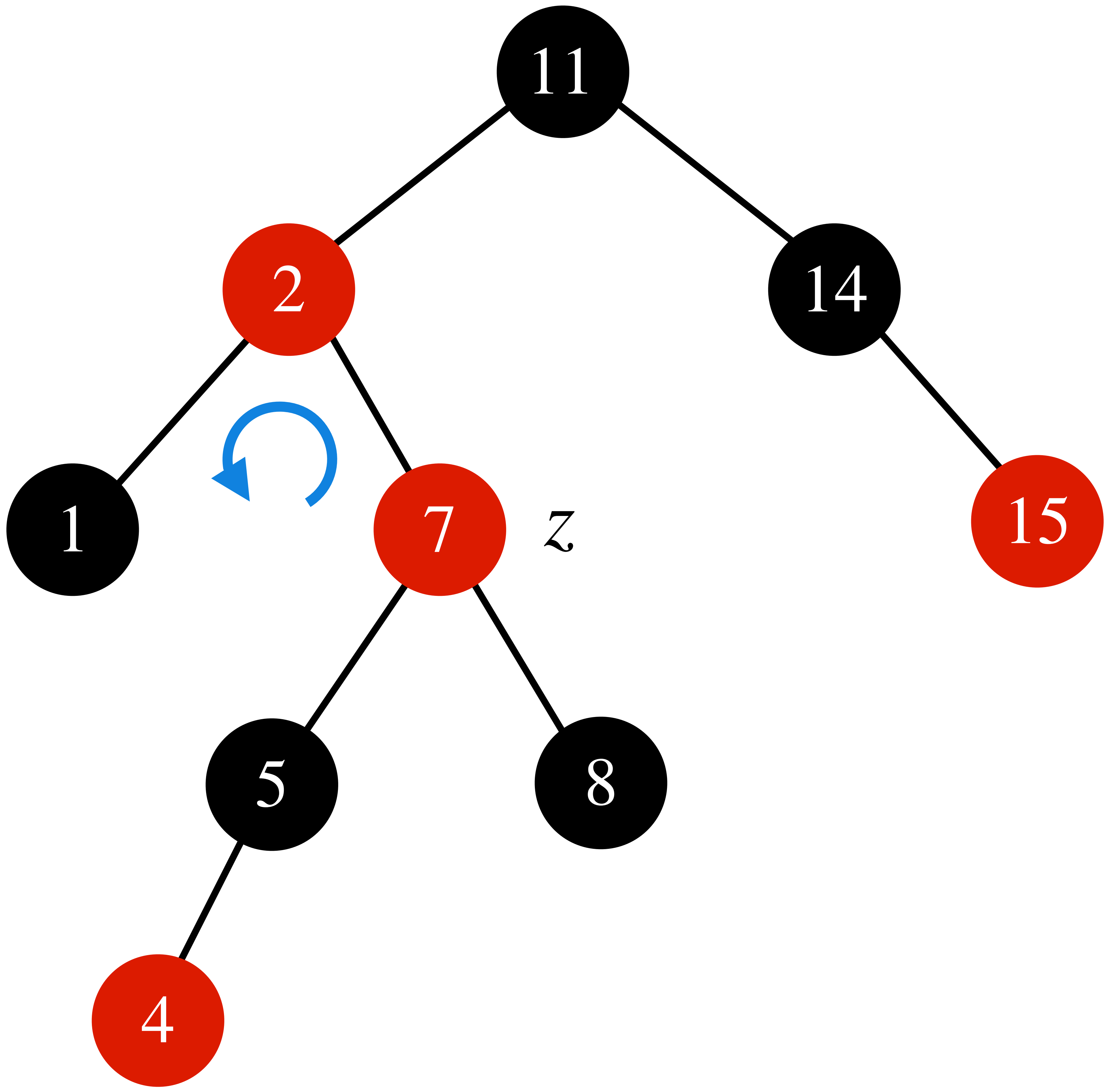

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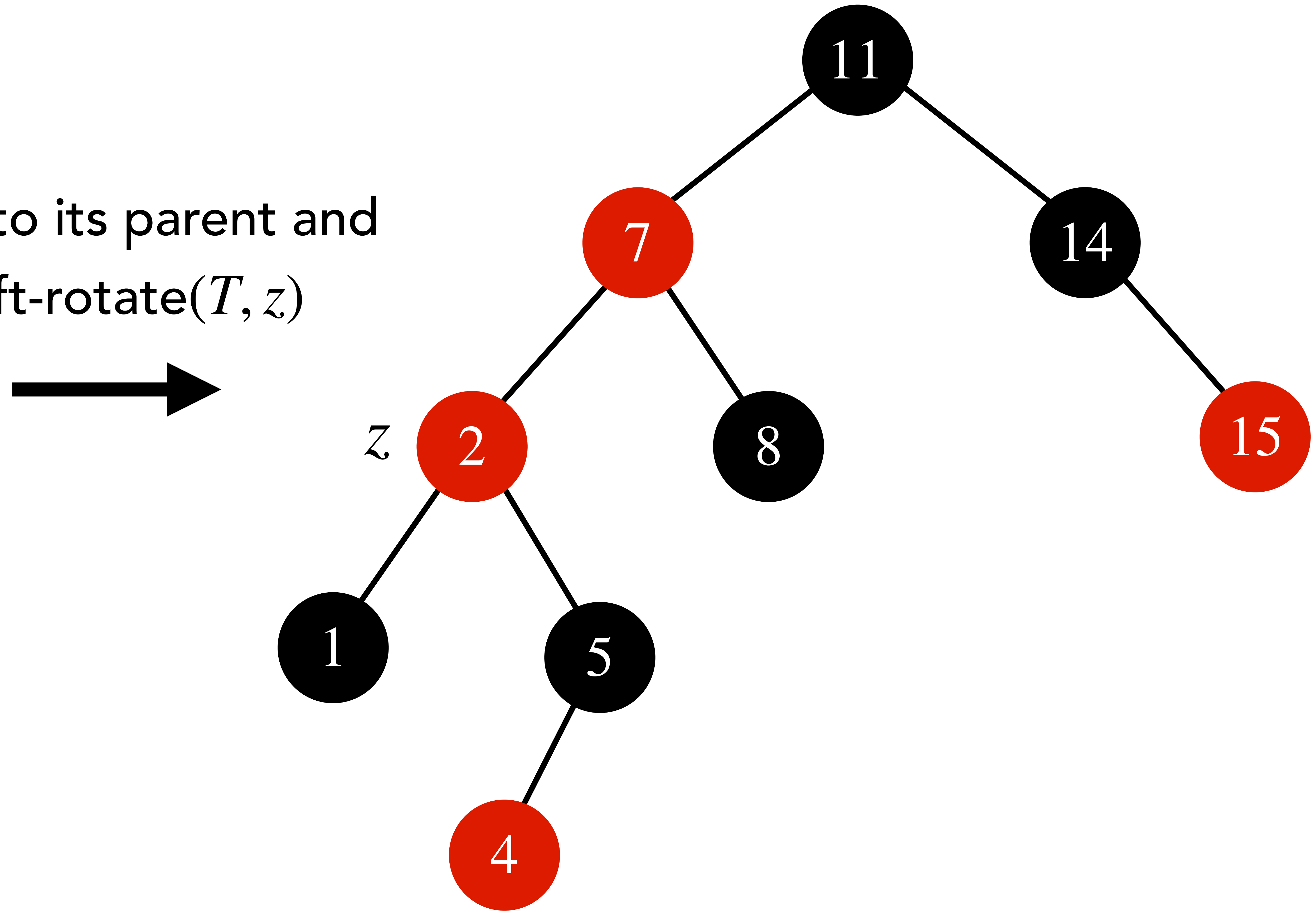
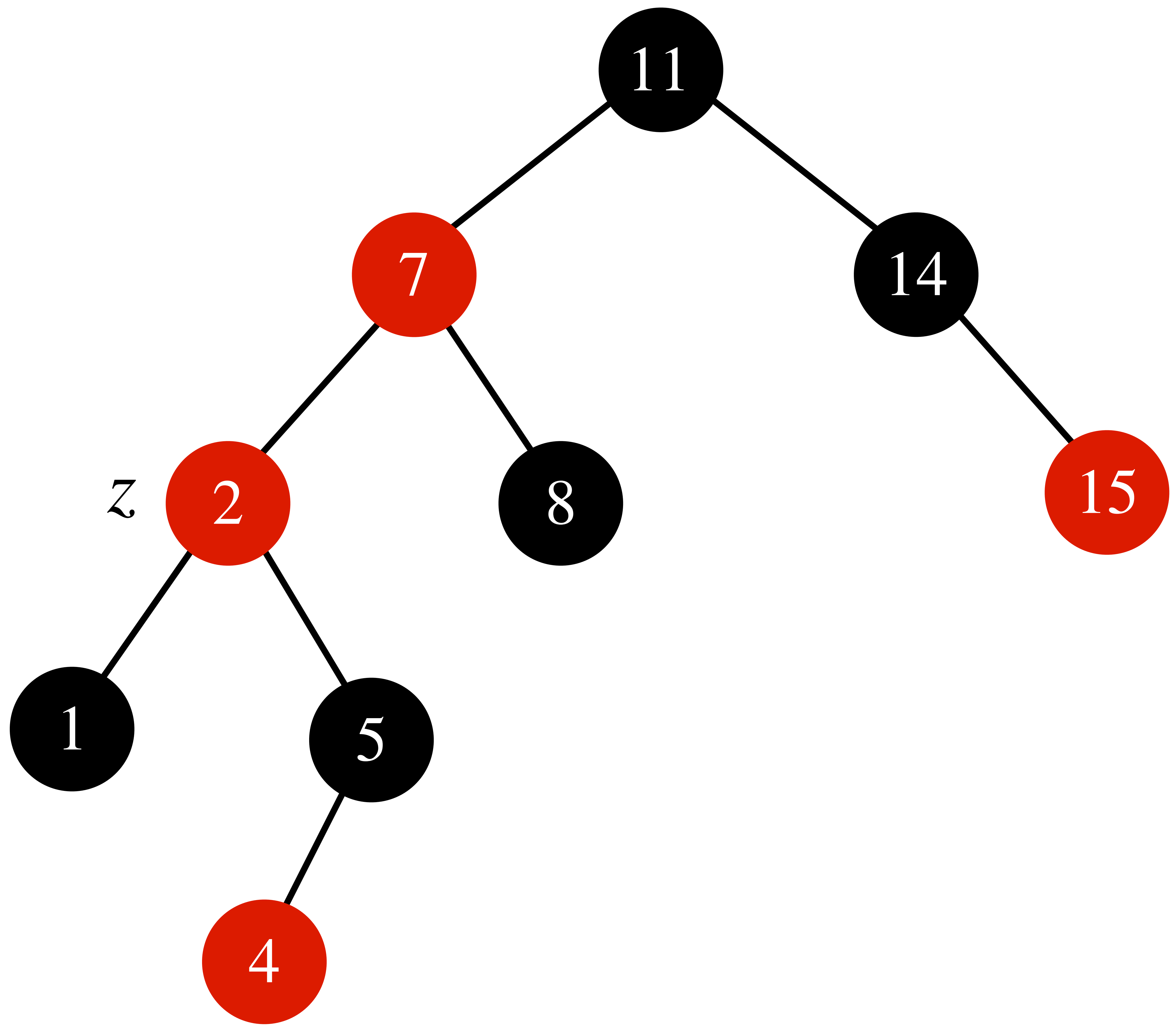


RB-Trees: Insertion Case 3

Case 3: z's uncle is black and z is a left child.



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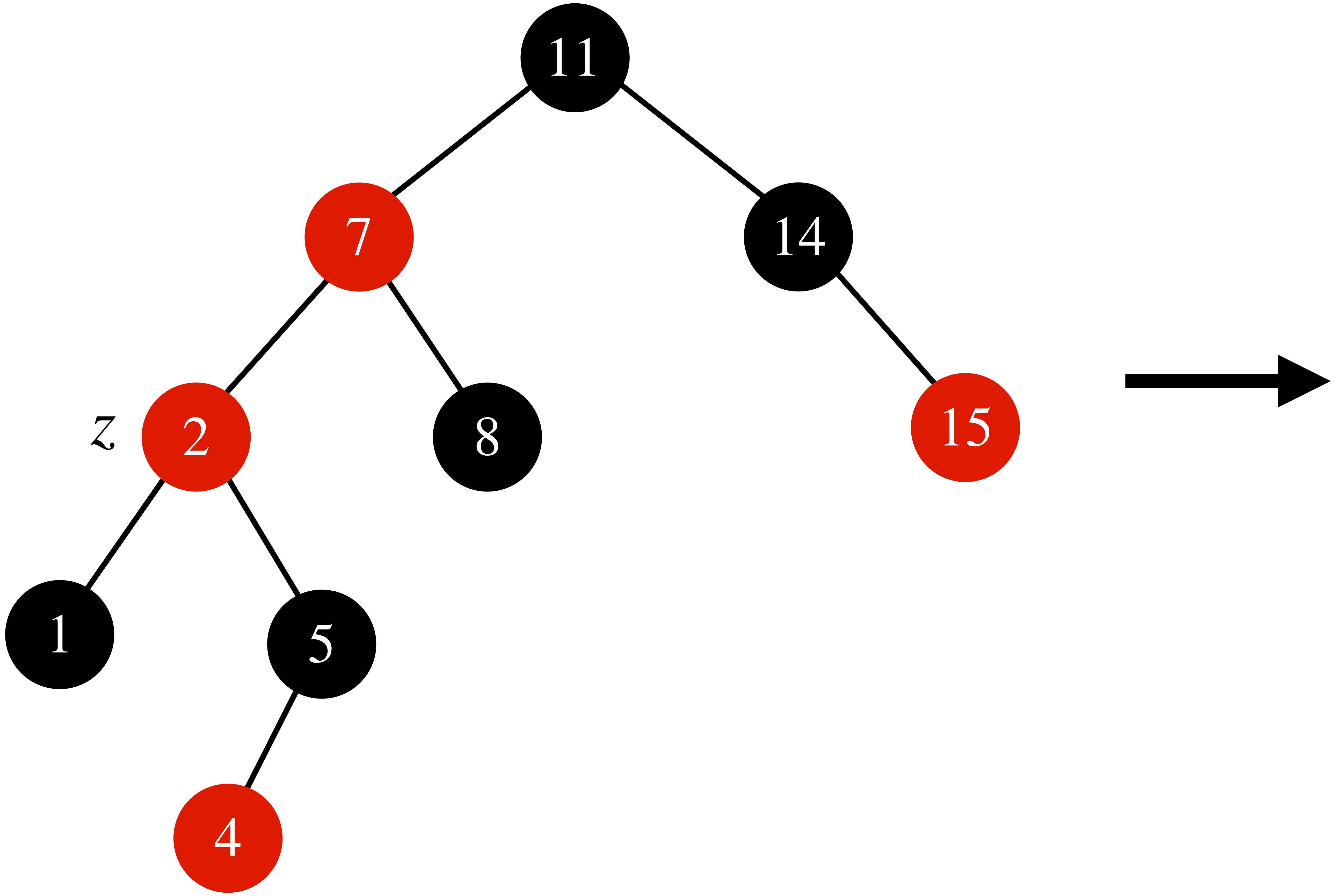

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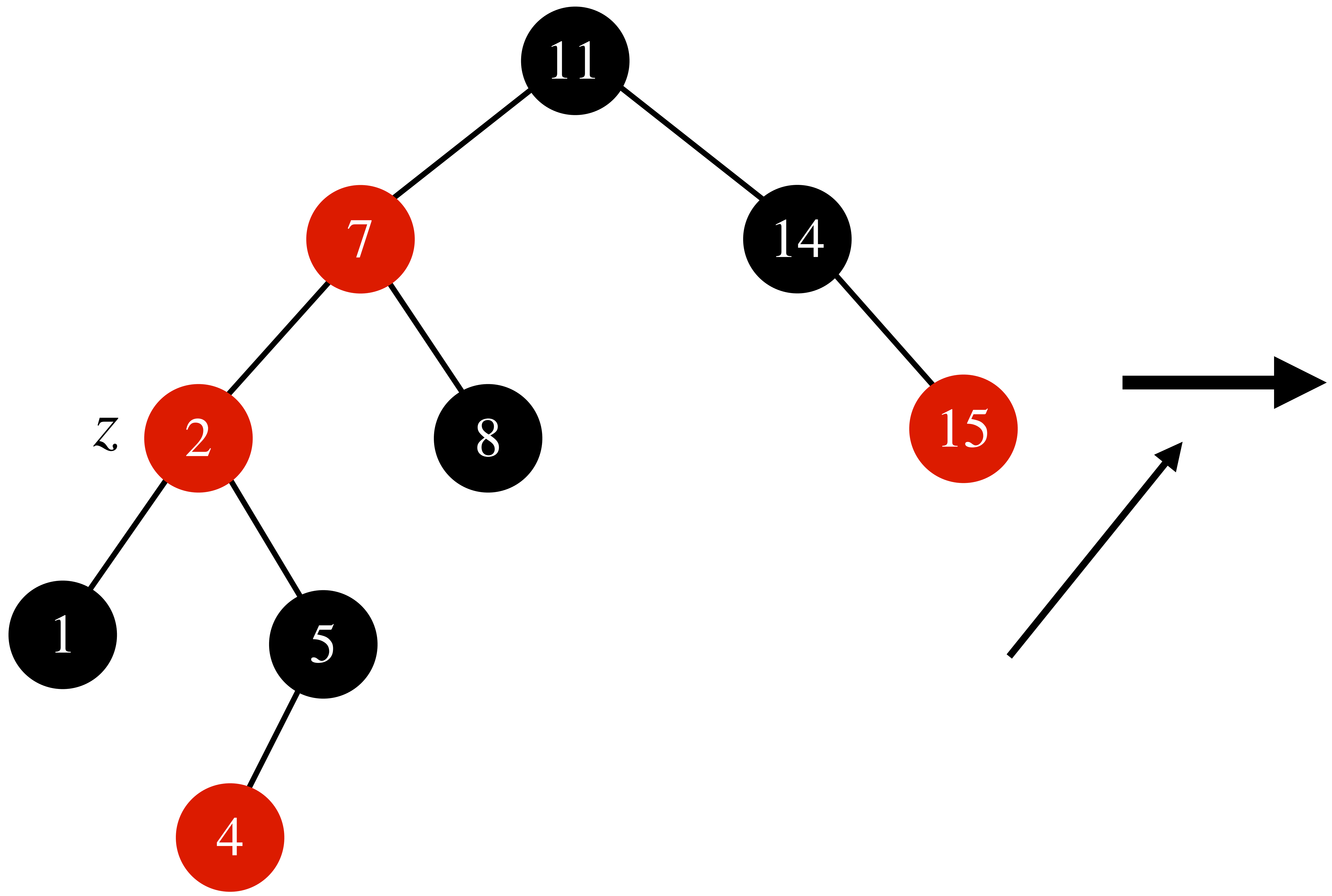

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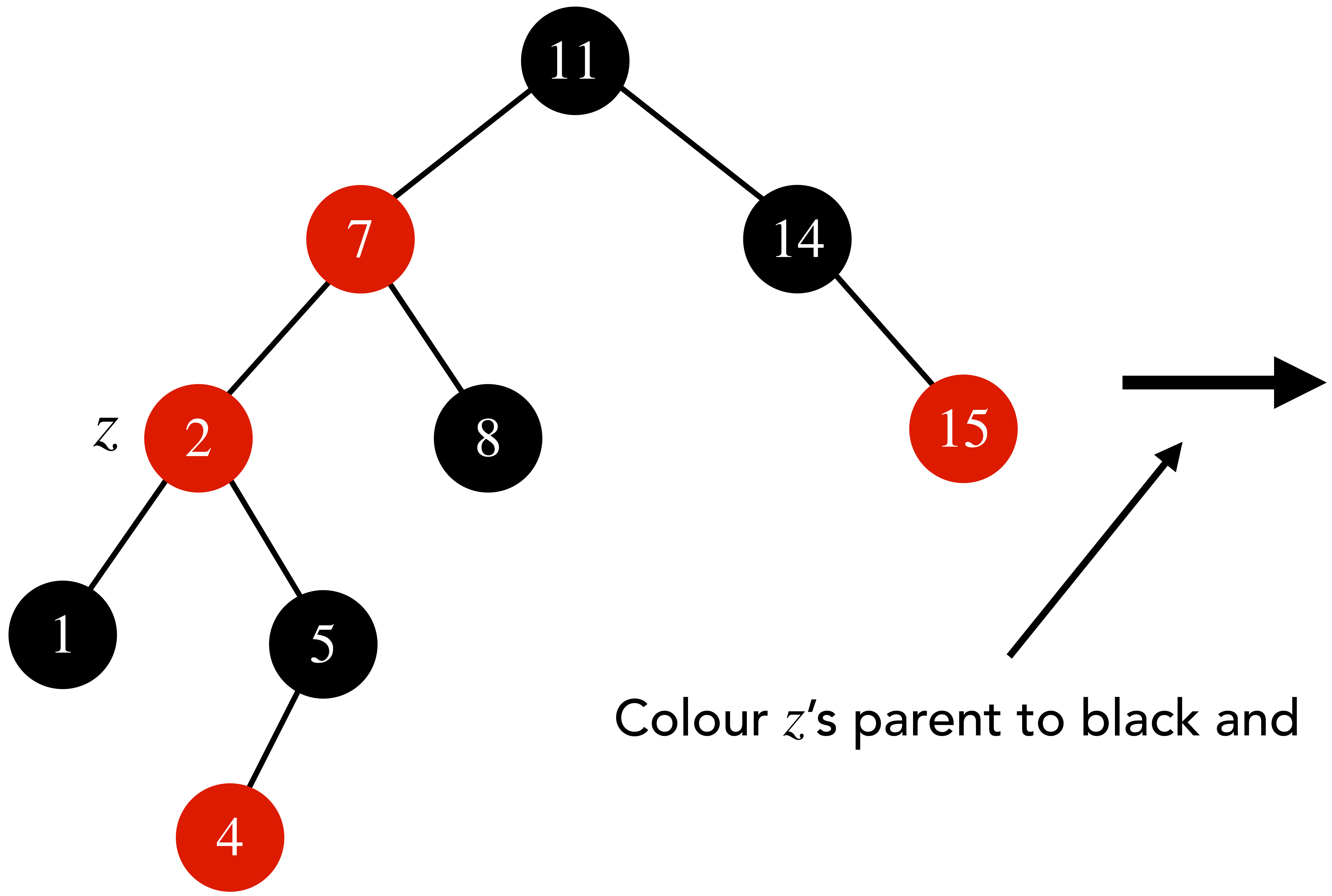

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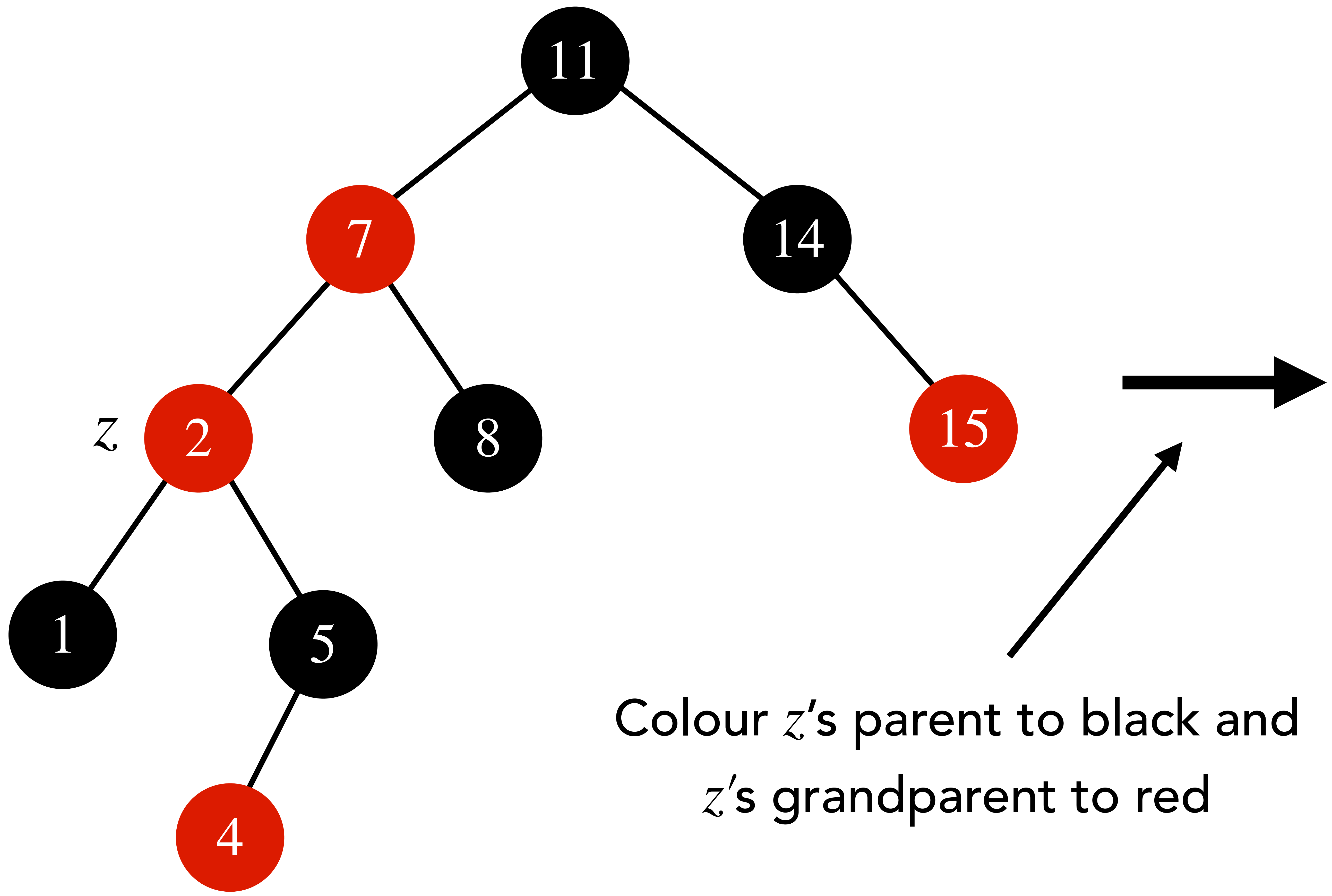

Colour z's parent to black and z's grandparent to red

RB-Trees: Insertion Case 3

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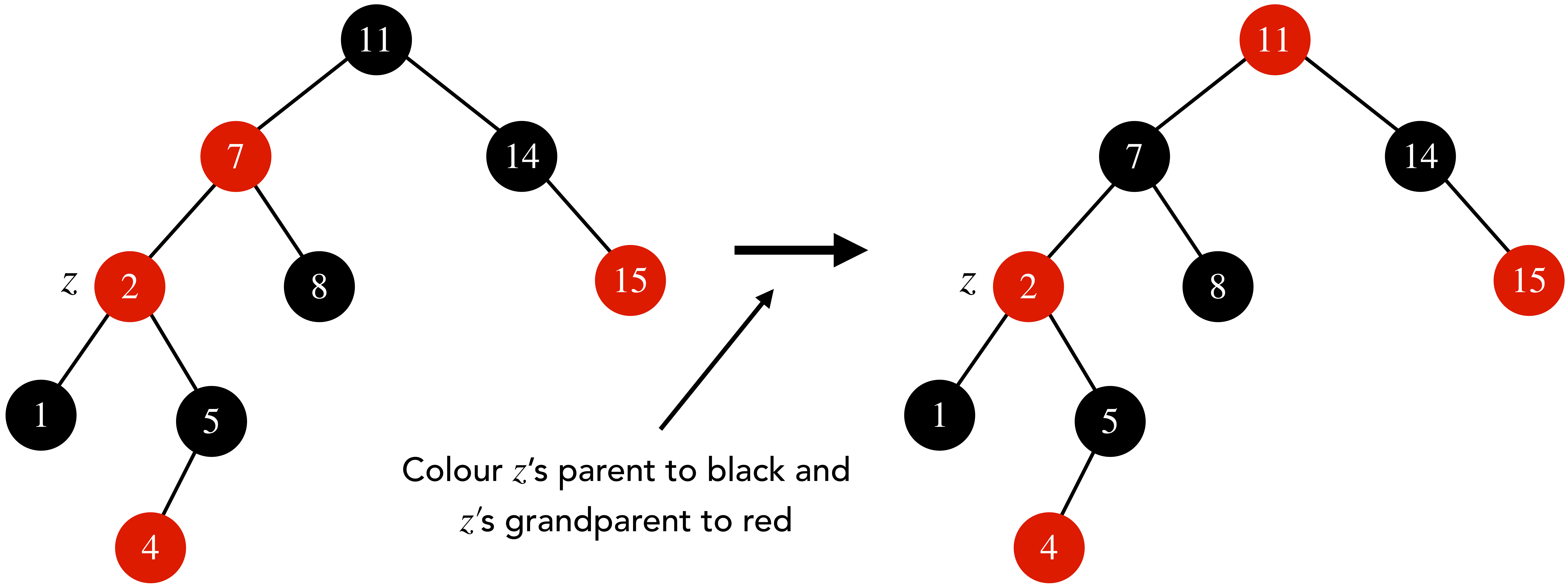

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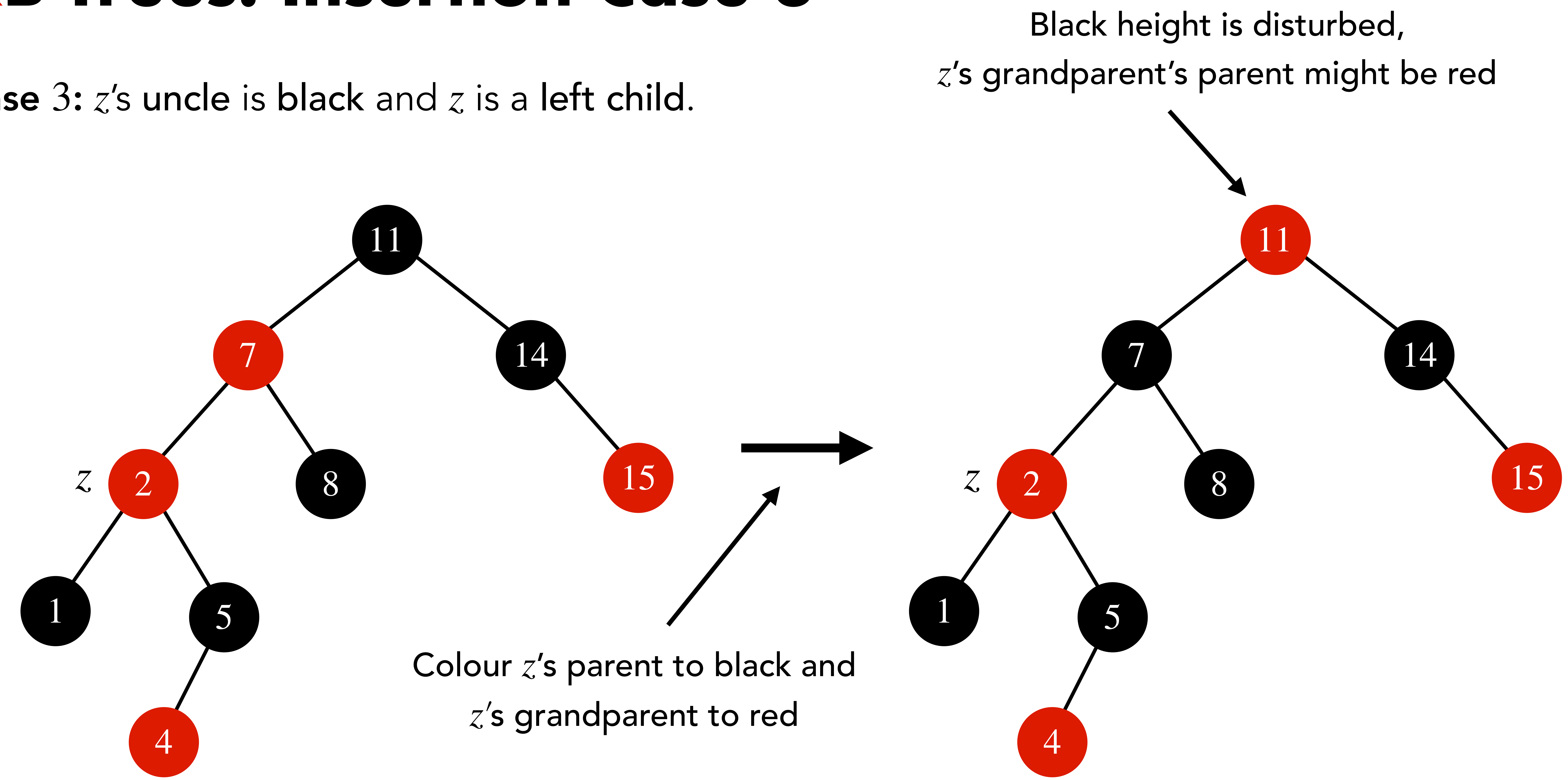
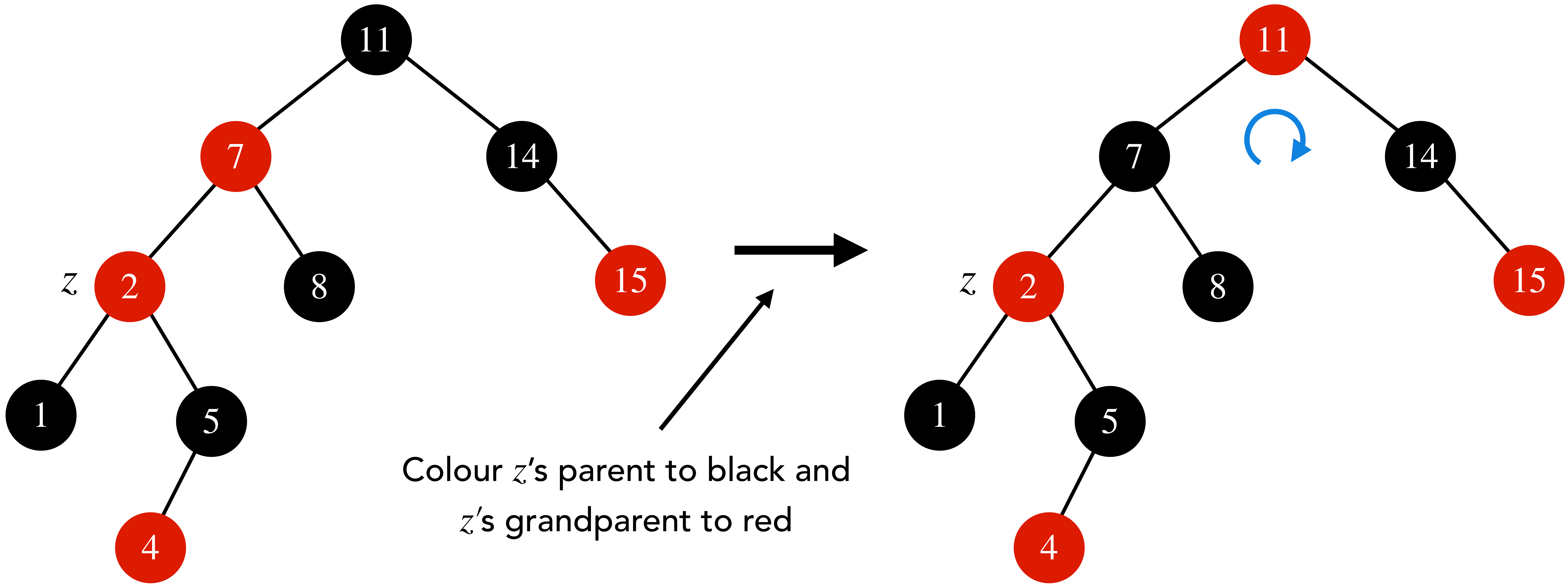


RB-Trees: Insertion Case 3

Case 3**:** z's uncle is black and z is a **left child**.

Black height is disturbed,
z's grandparent's parent might be red



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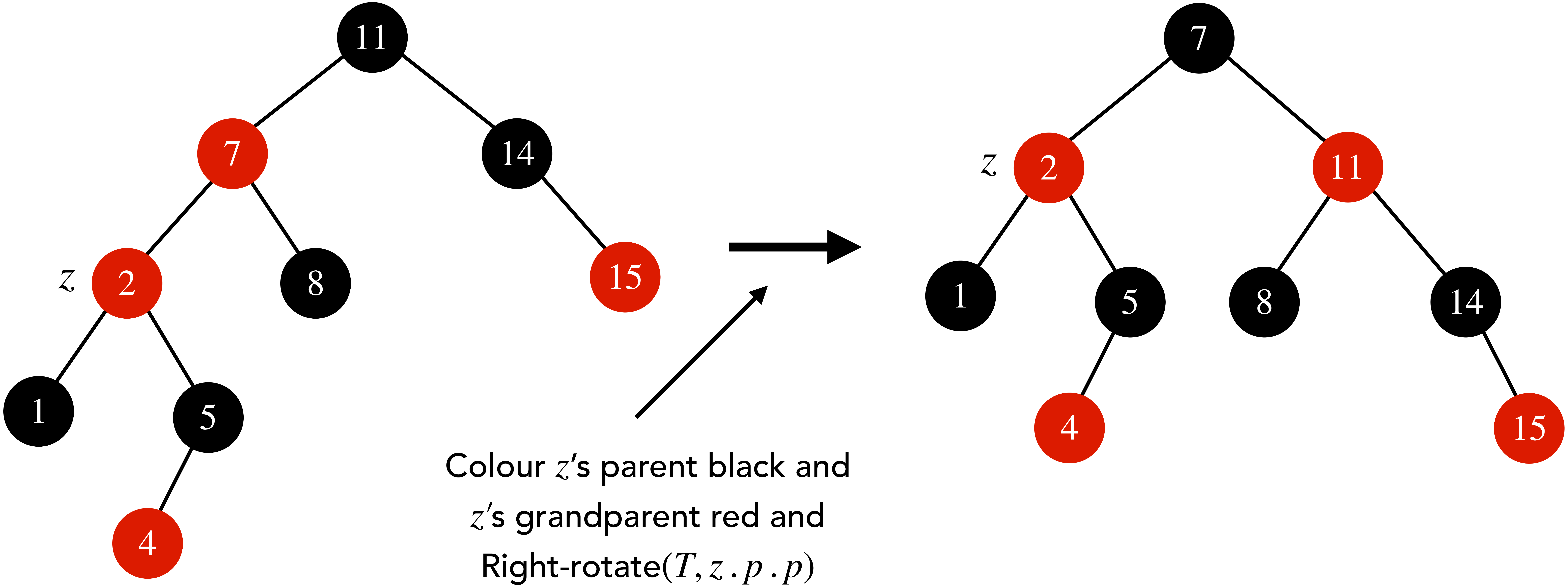

Colour z's parent black and z's grandparent red and Right-rotate$(T, z.p.p)$

RB-Trees: Deletion

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Two stages of deletion:

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Let's recall deletion in a BST and spot special nodes, y and x.

RB-Trees: Deletion

Two stages of deletion:

- Delete the node as we do in a BST.
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Let's recall deletion in a BST and spot special nodes, y and x.

y will be the node we will "actually" be taking out
and whether fix ups are require will depend on the colour of y

RB-Trees: Deletion

Two stages of deletion:

- Delete the node as we do in a BST.
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Let's recall deletion in a BST and spot special nodes, y and x.

Fix ups will start from x after removing y

Recall Deletion in BSTs

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Easy

Not so easy

RB-Trees: Deletion

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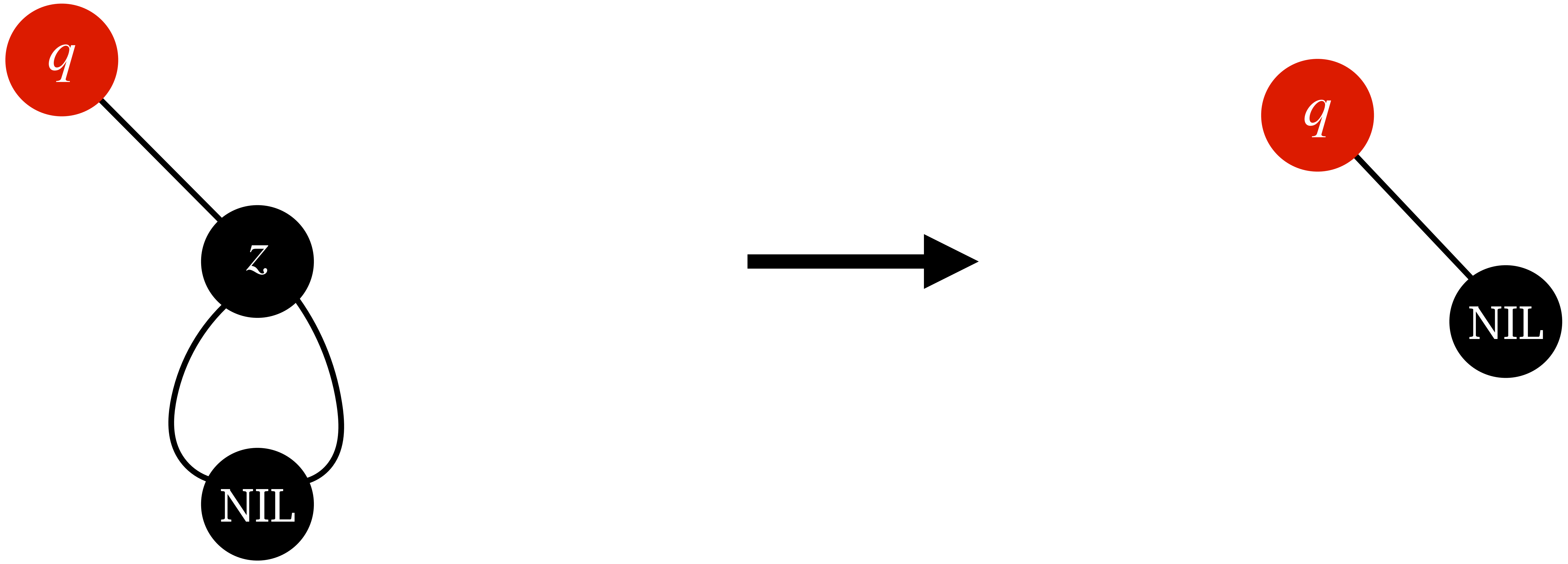

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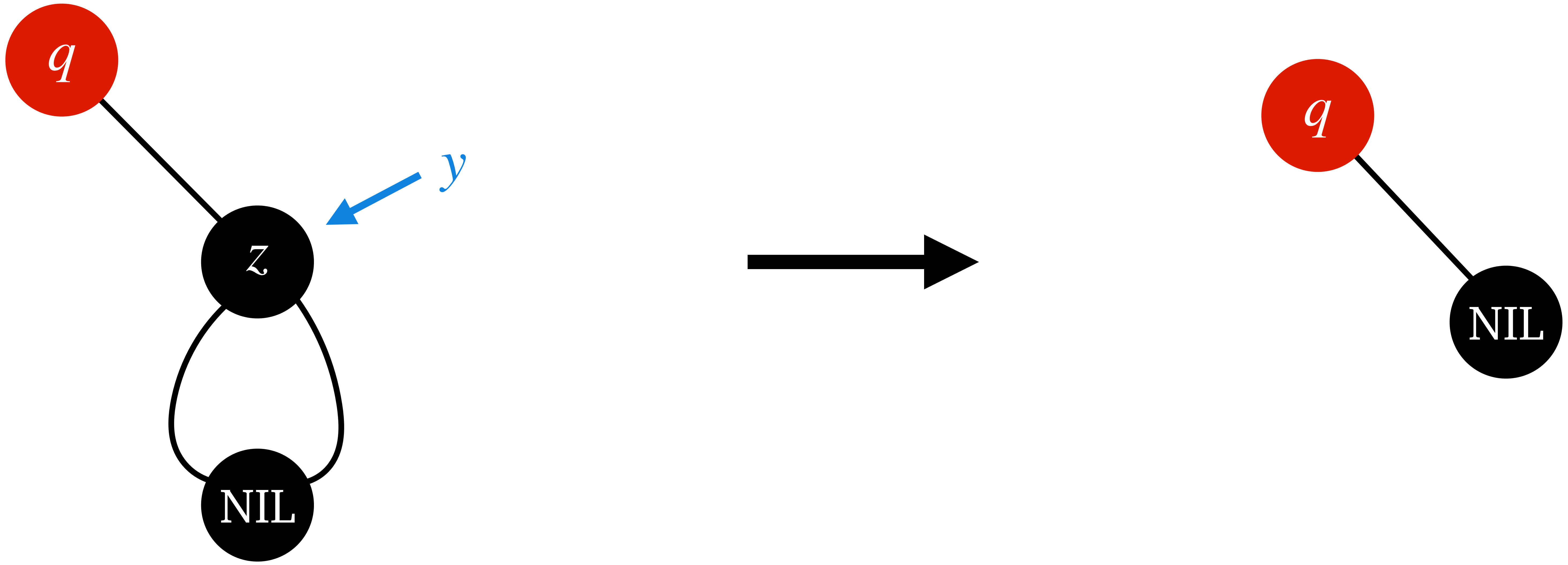

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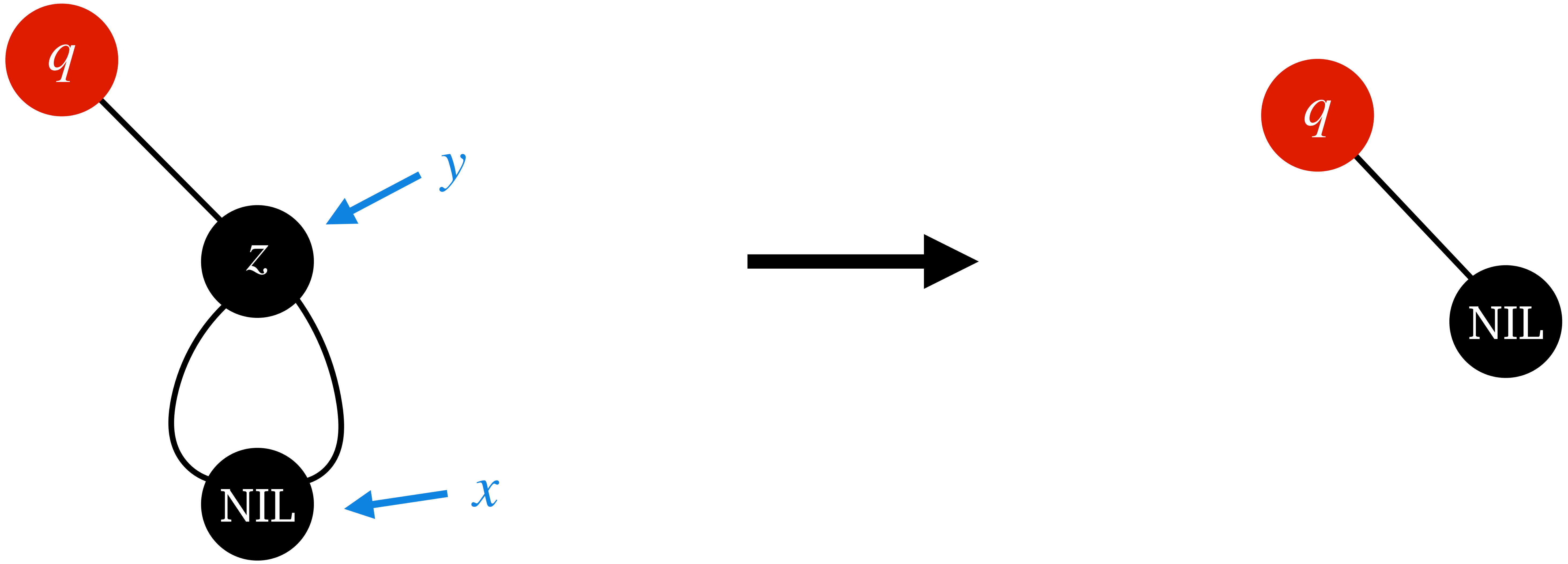

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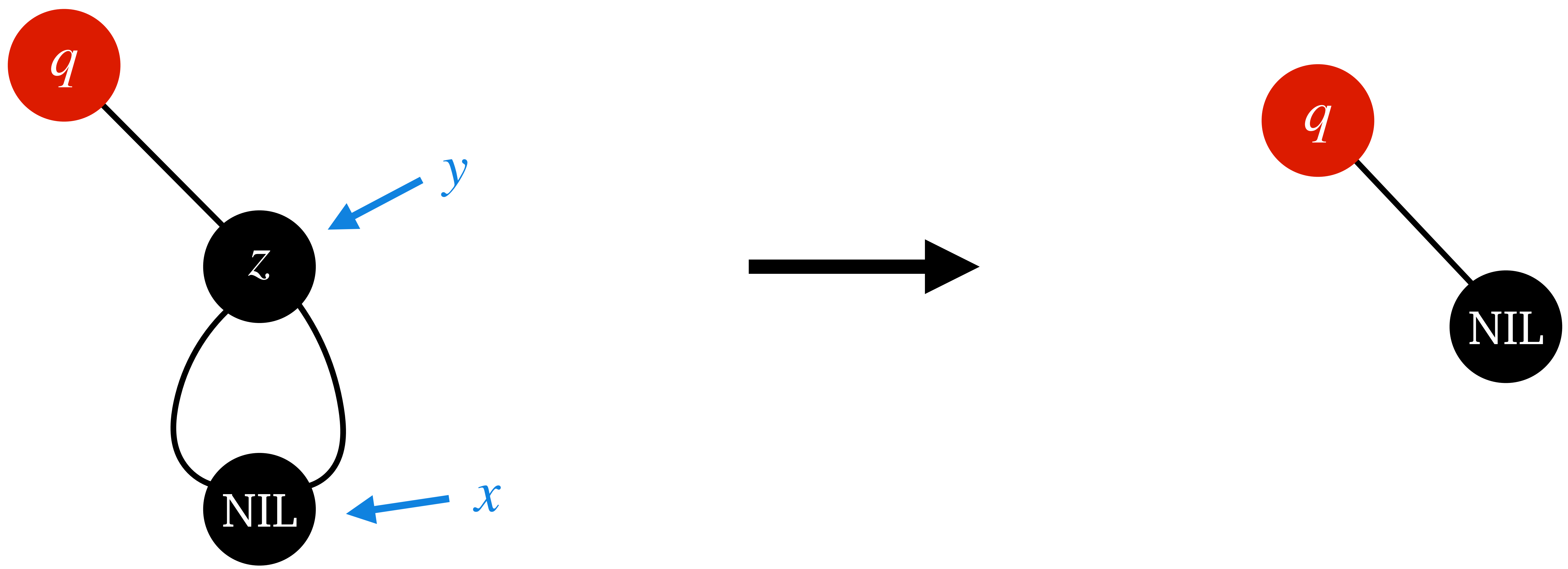
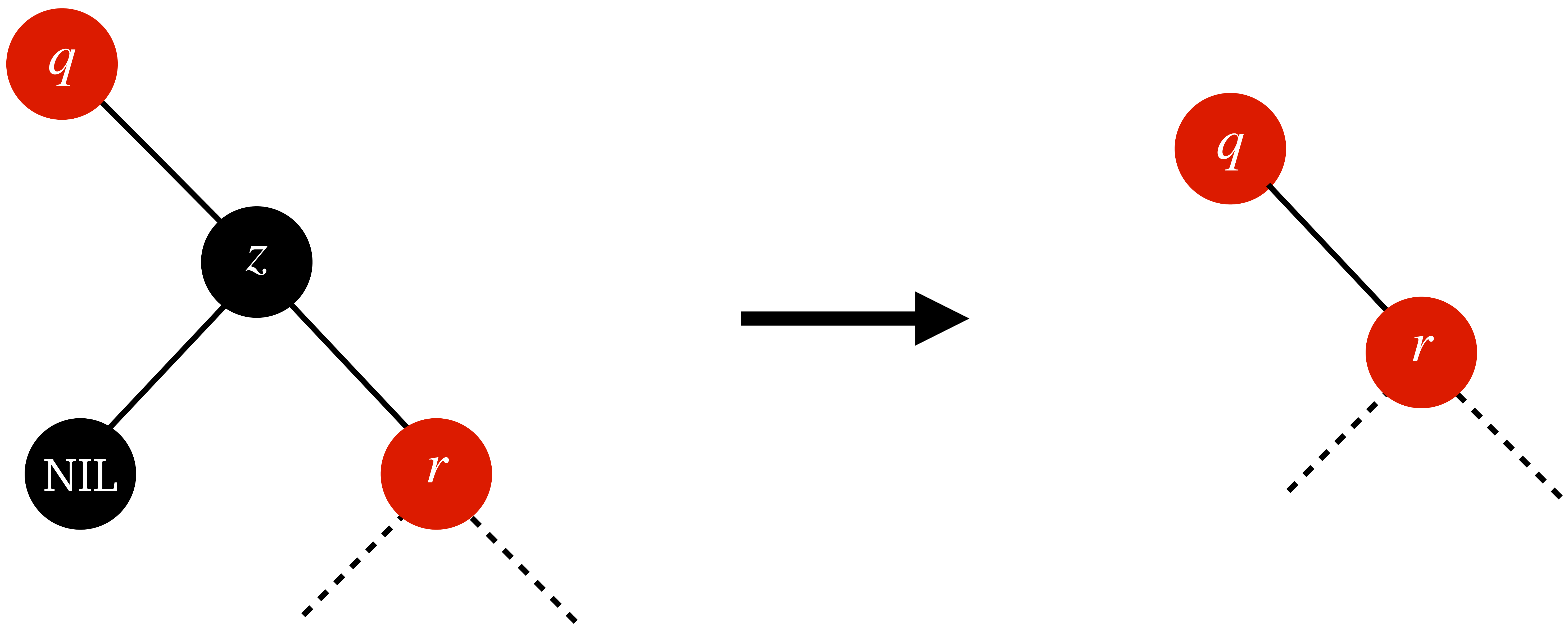


Note: In this case, y is z and x is NIL.

RB-Trees: Deletion

Case 2: z has one (non-NIL) child. (WLOG assume z is a right child.)



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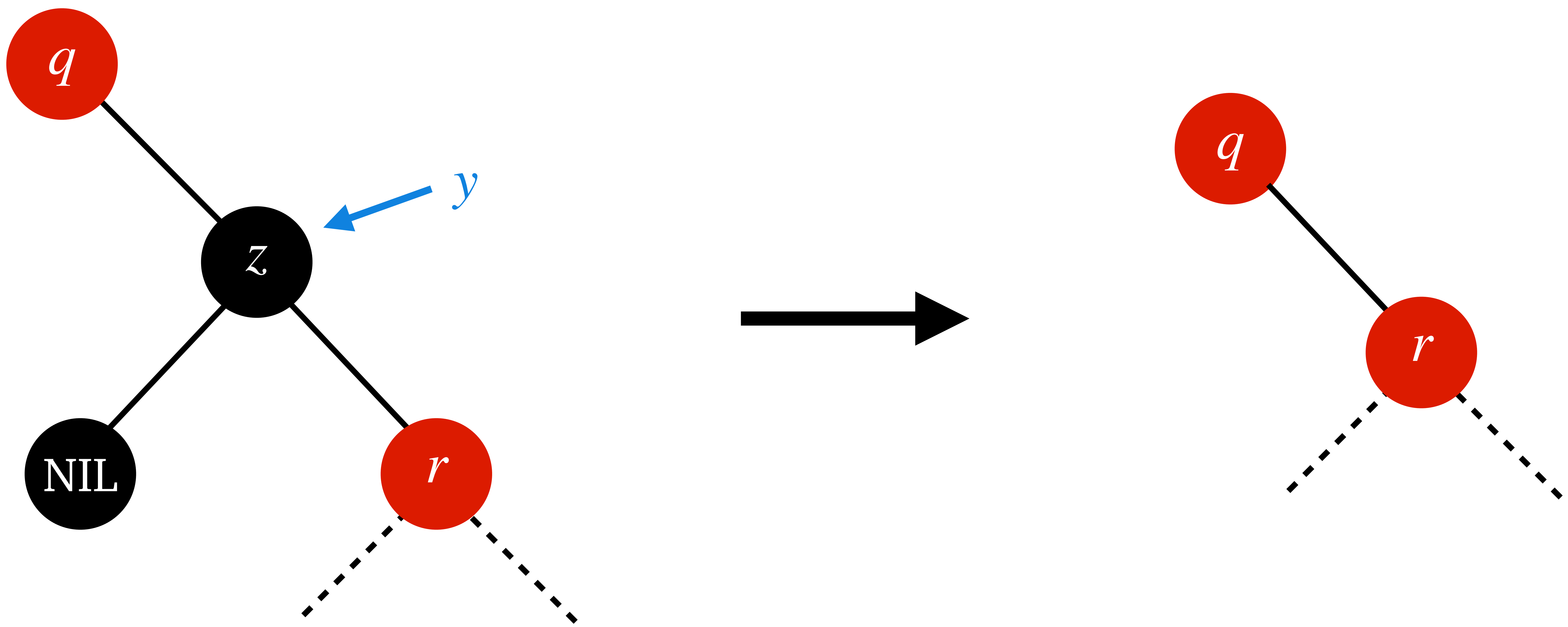

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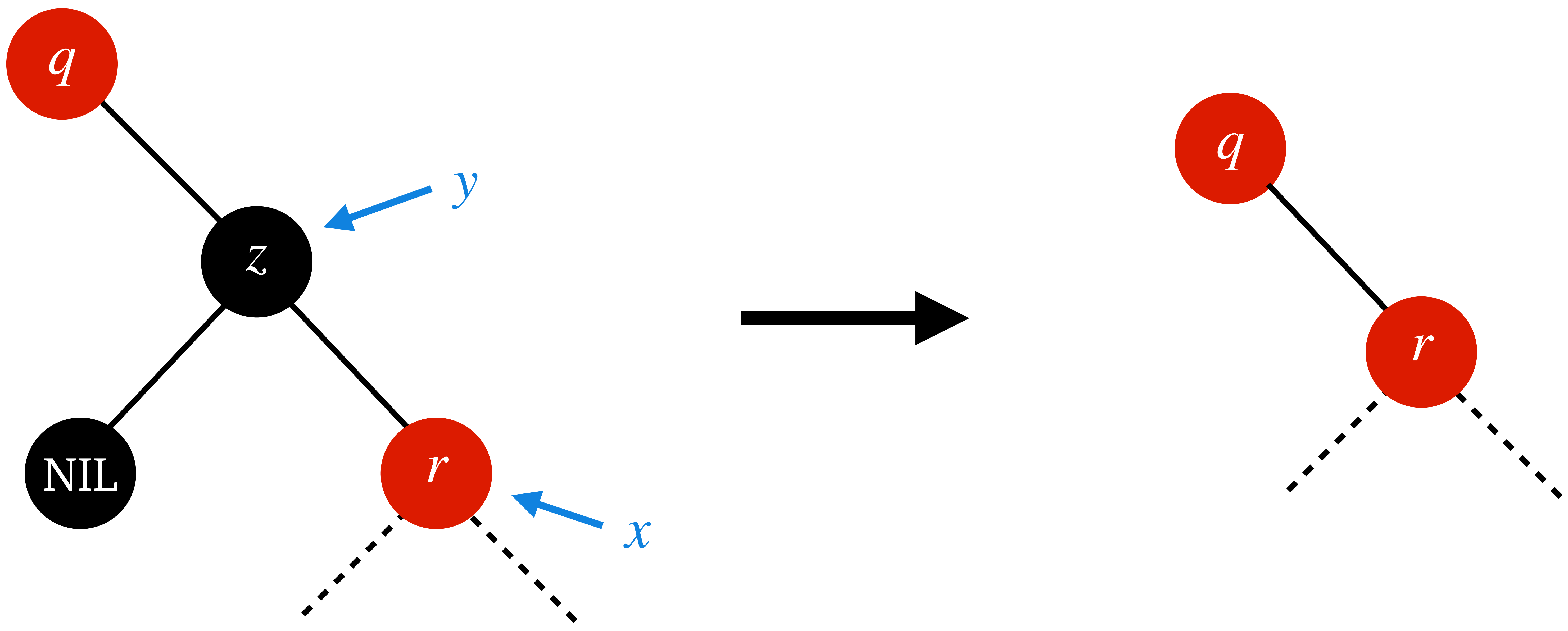

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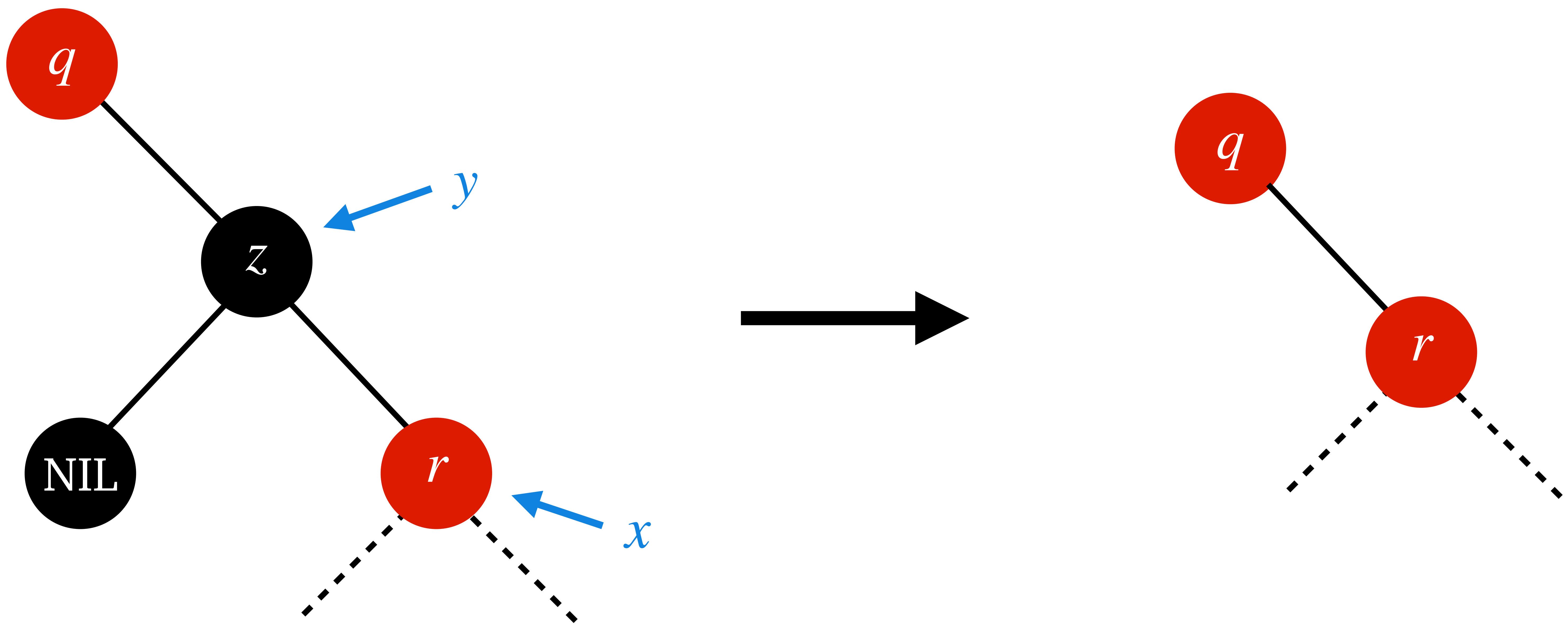
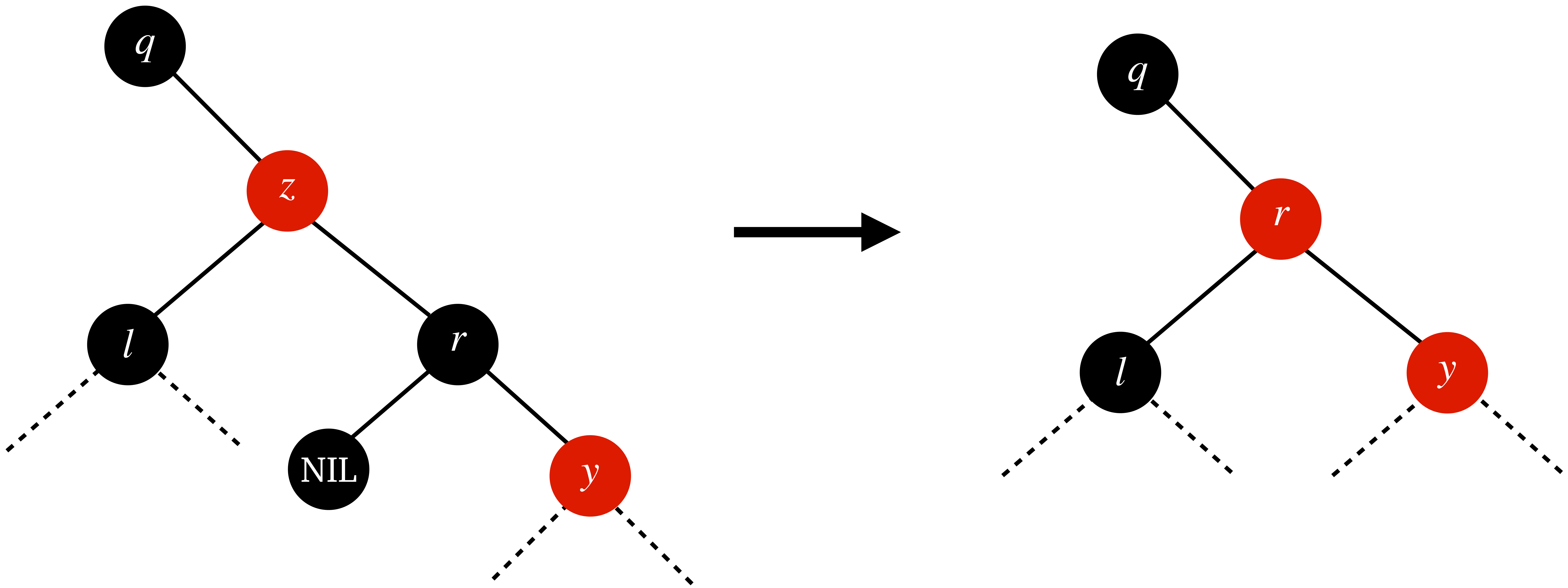


Note: In this case, y is z and x is the only child of z.

RB-Trees: Deletion

Case 3**a**: z has two (non-NIL) children where its right child has no left child.



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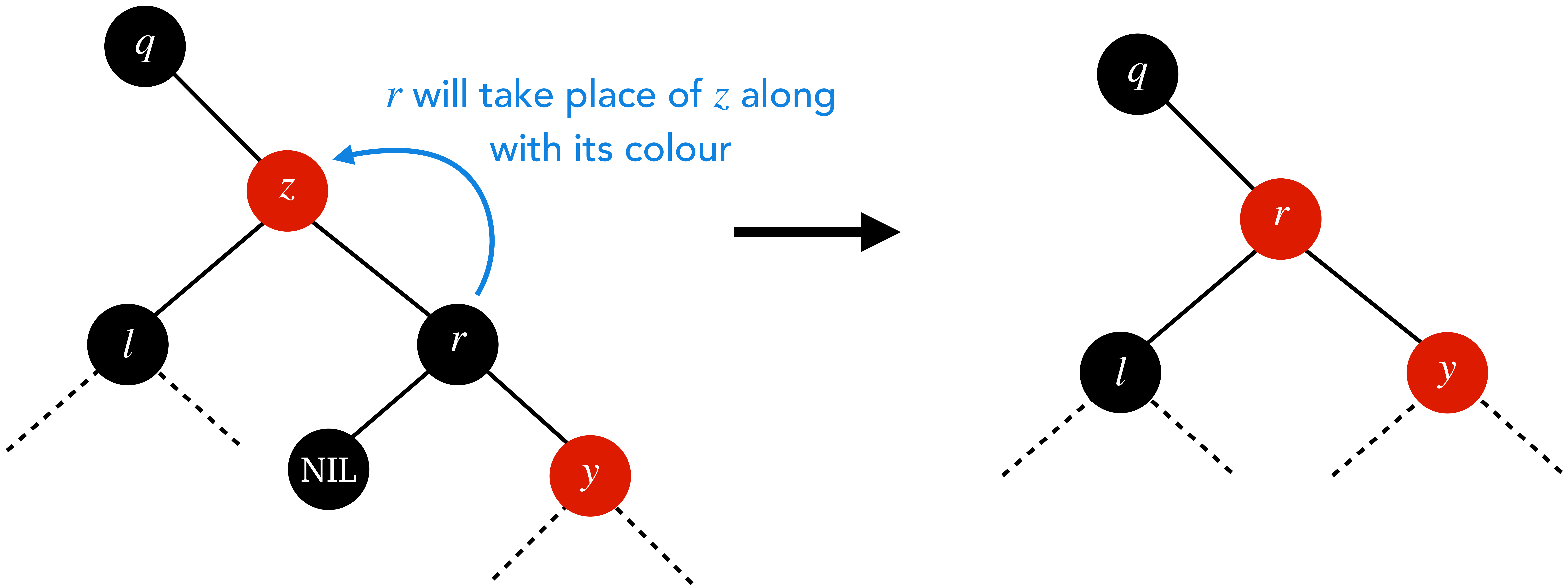

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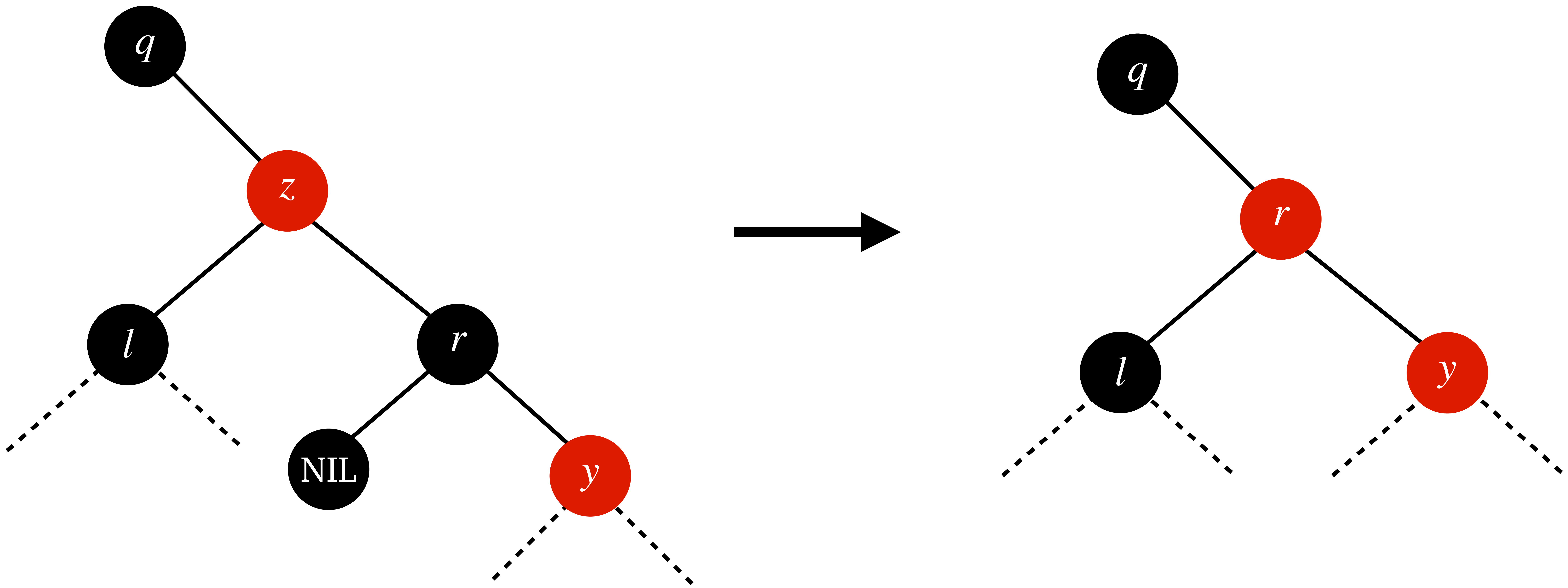

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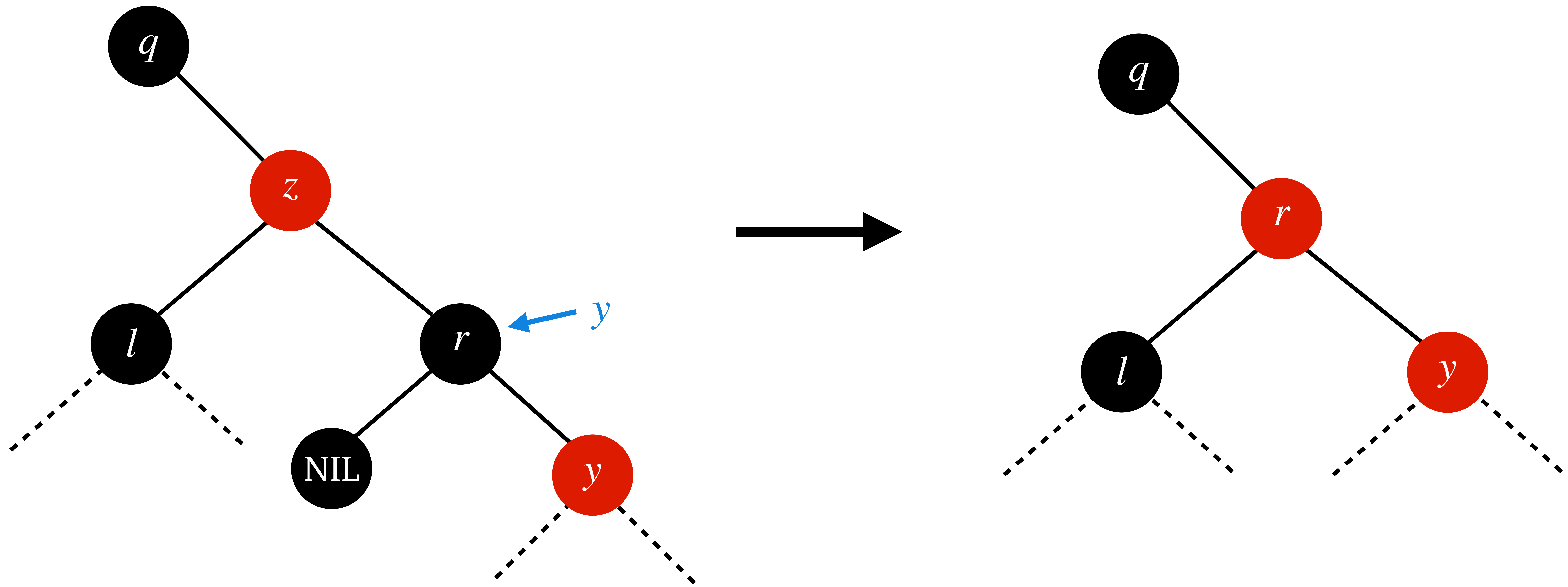

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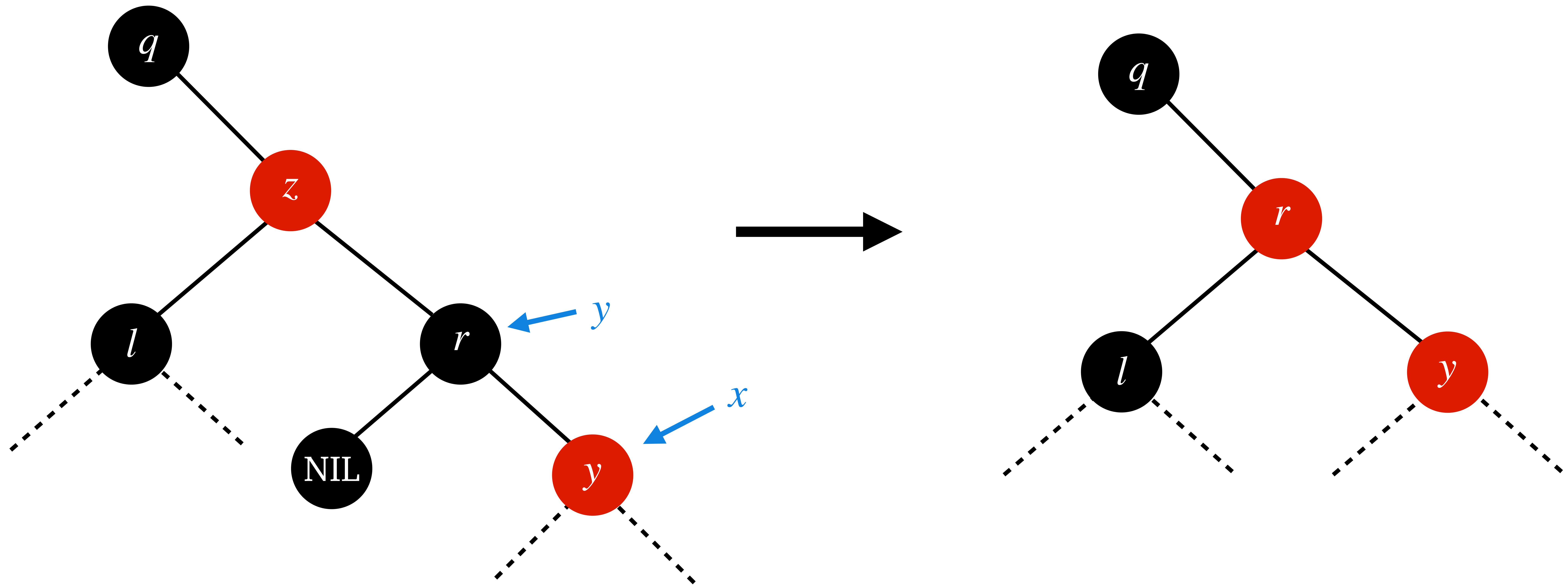
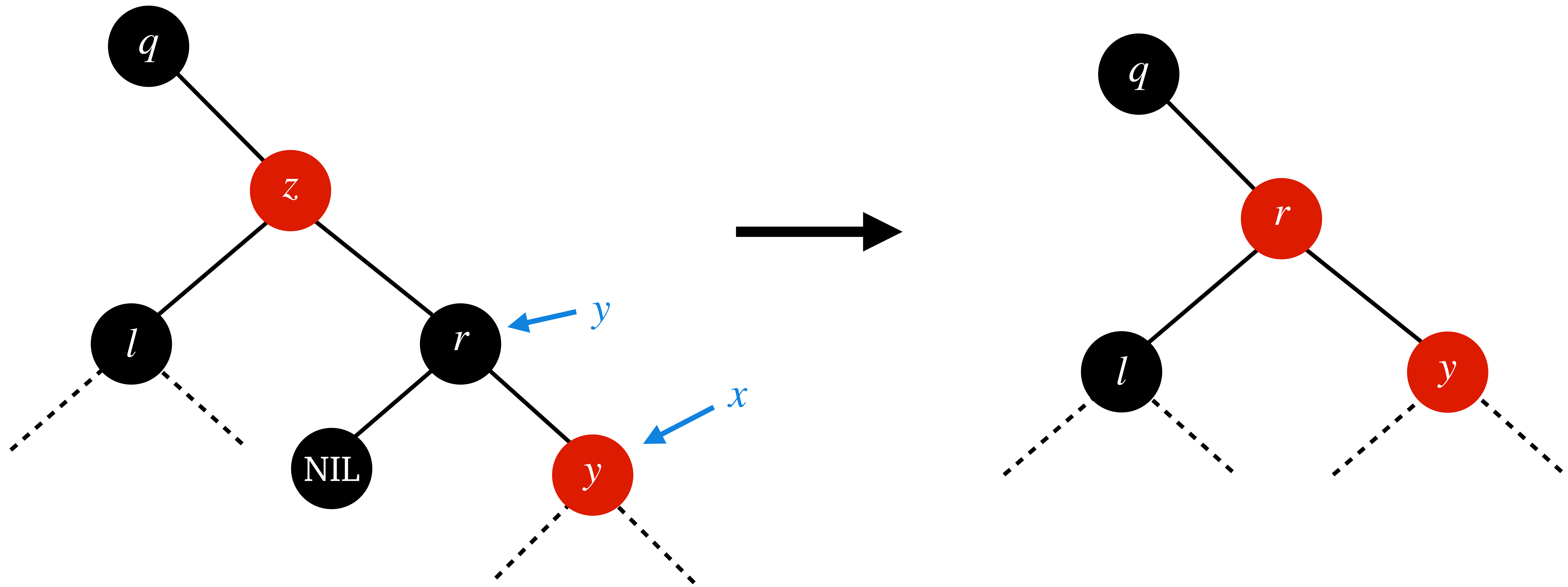


RB-Trees: Deletion

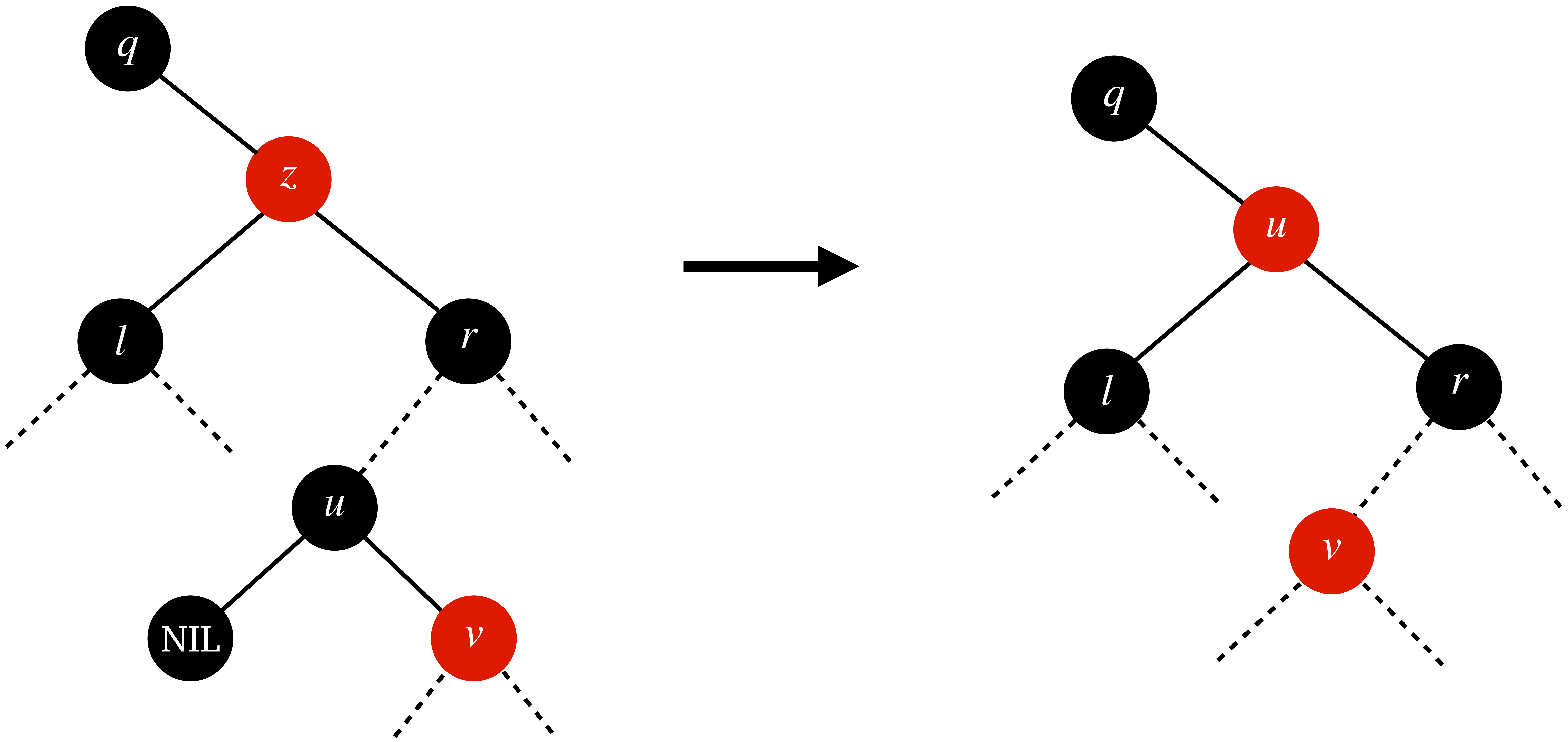
Case 3a: z has two (non-NIL) children where its right child has no left child.



Note: In this case, y is the successor of z and x is either NIL or the only child of y.

RB-Trees: Deletion

Case 3**b**: z has two (non-NIL) children where its right child has a left child.



RB-Trees: Deletion

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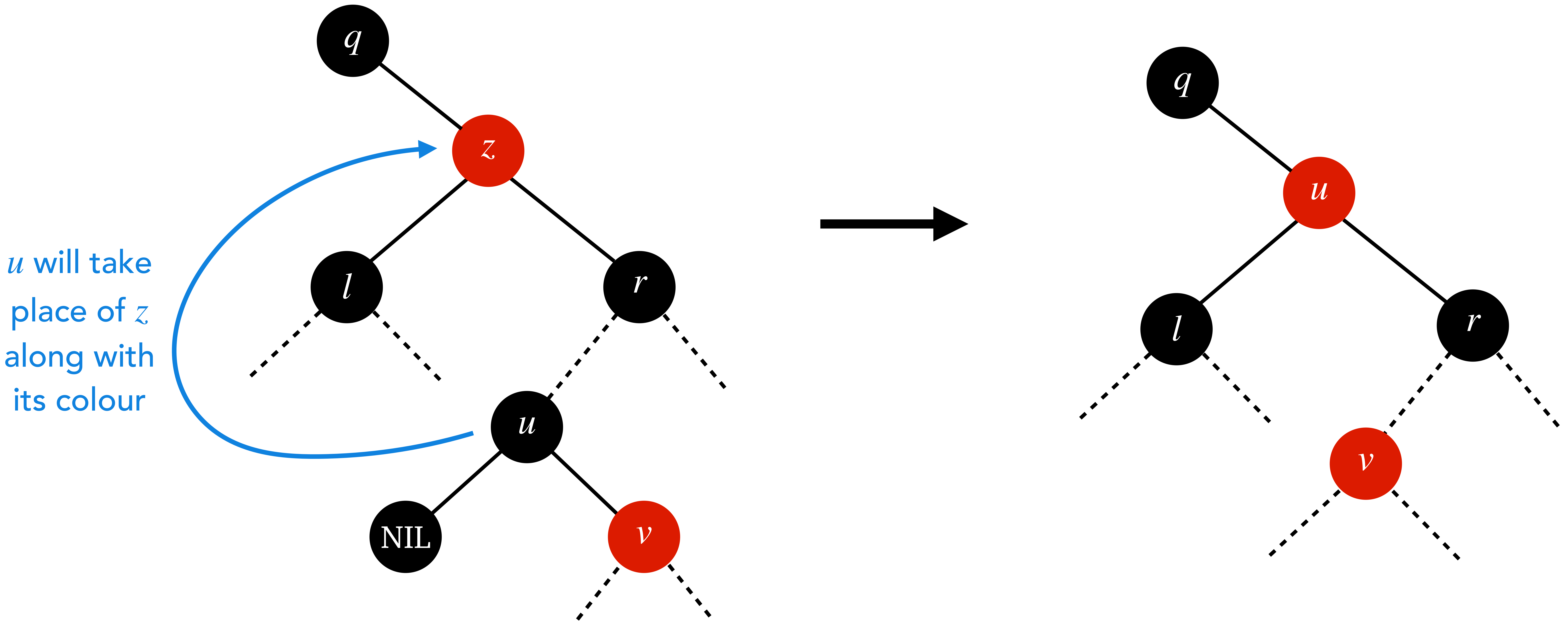

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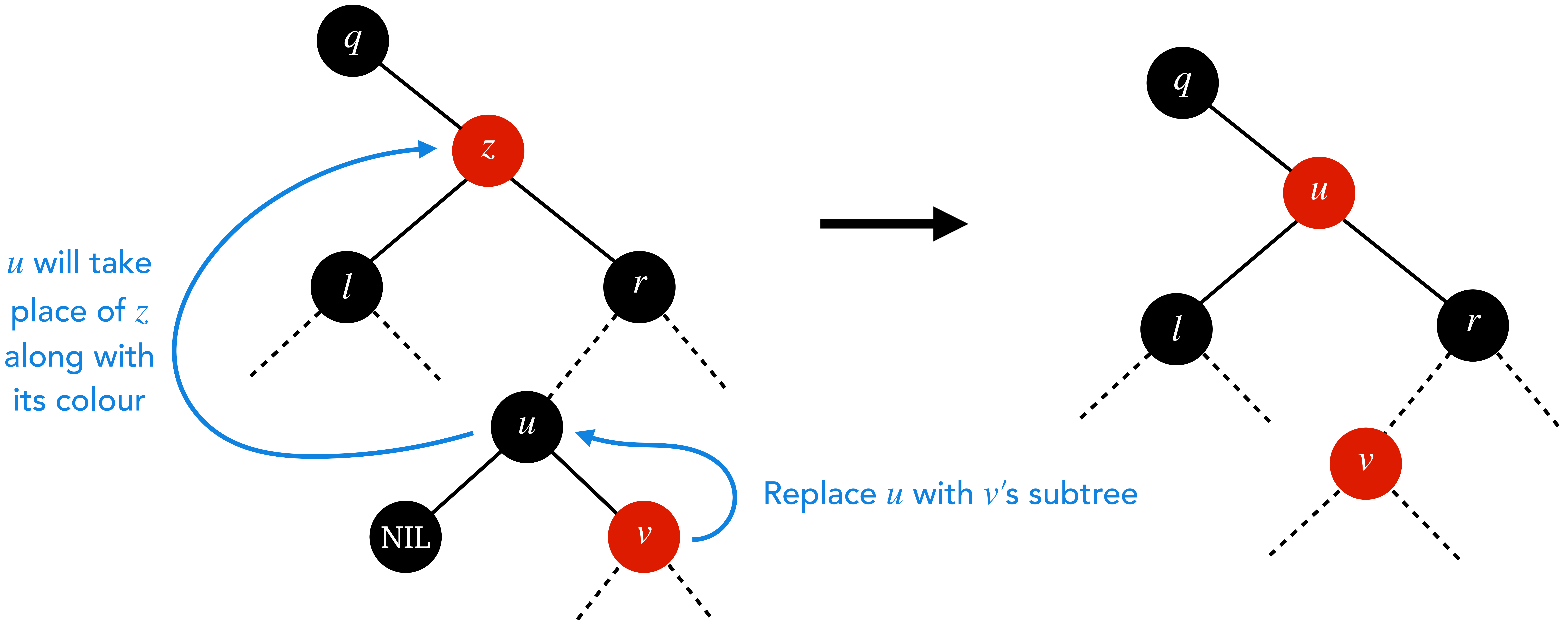

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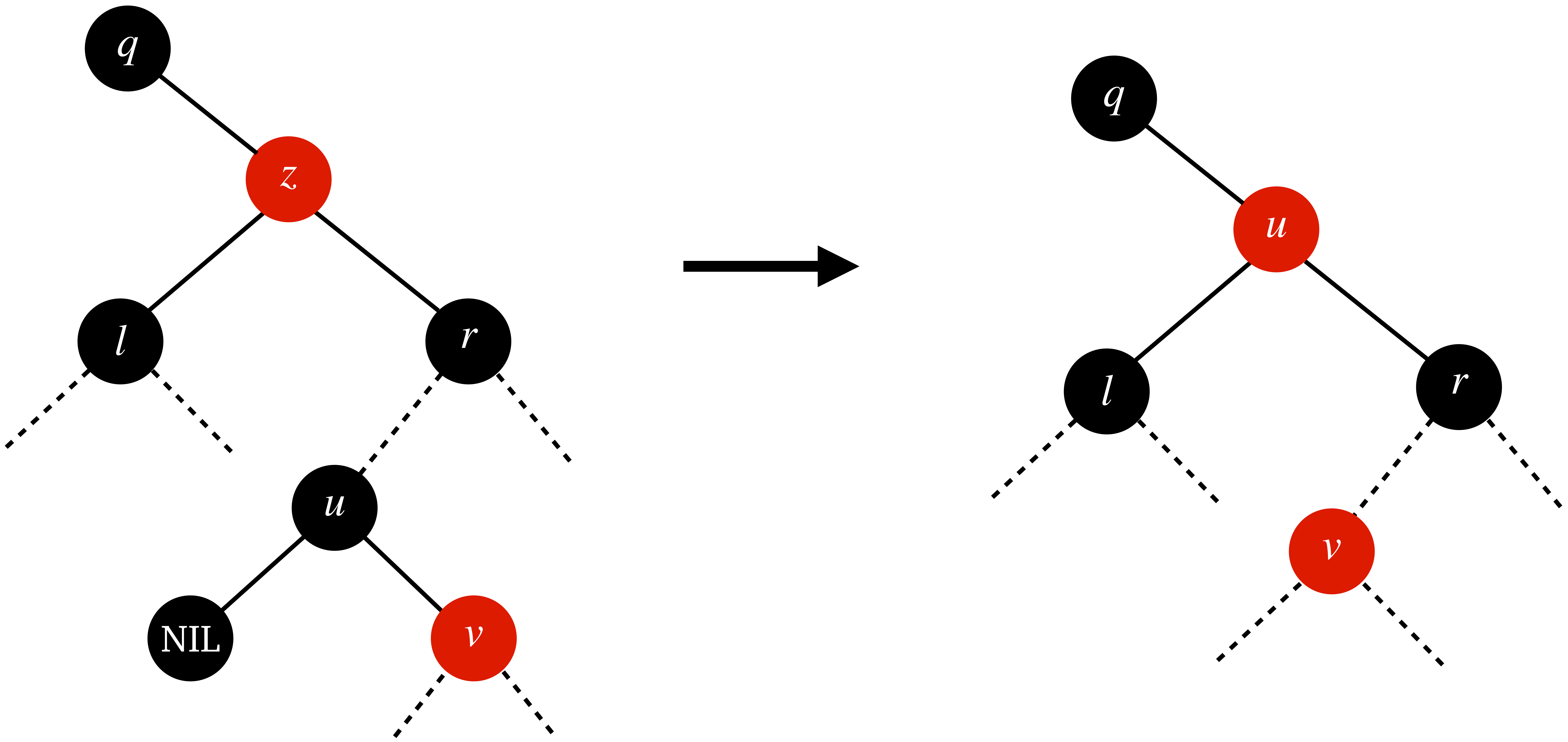

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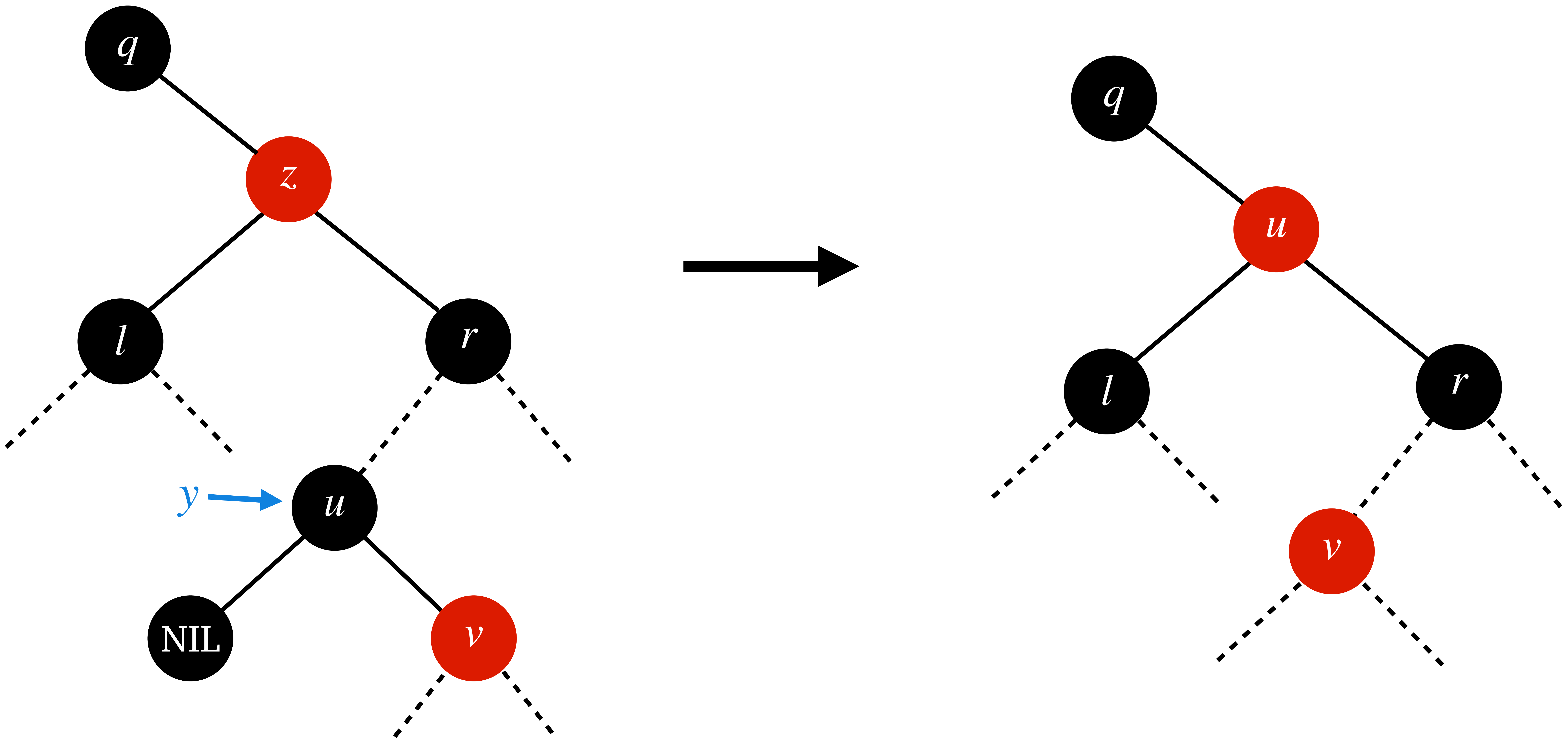

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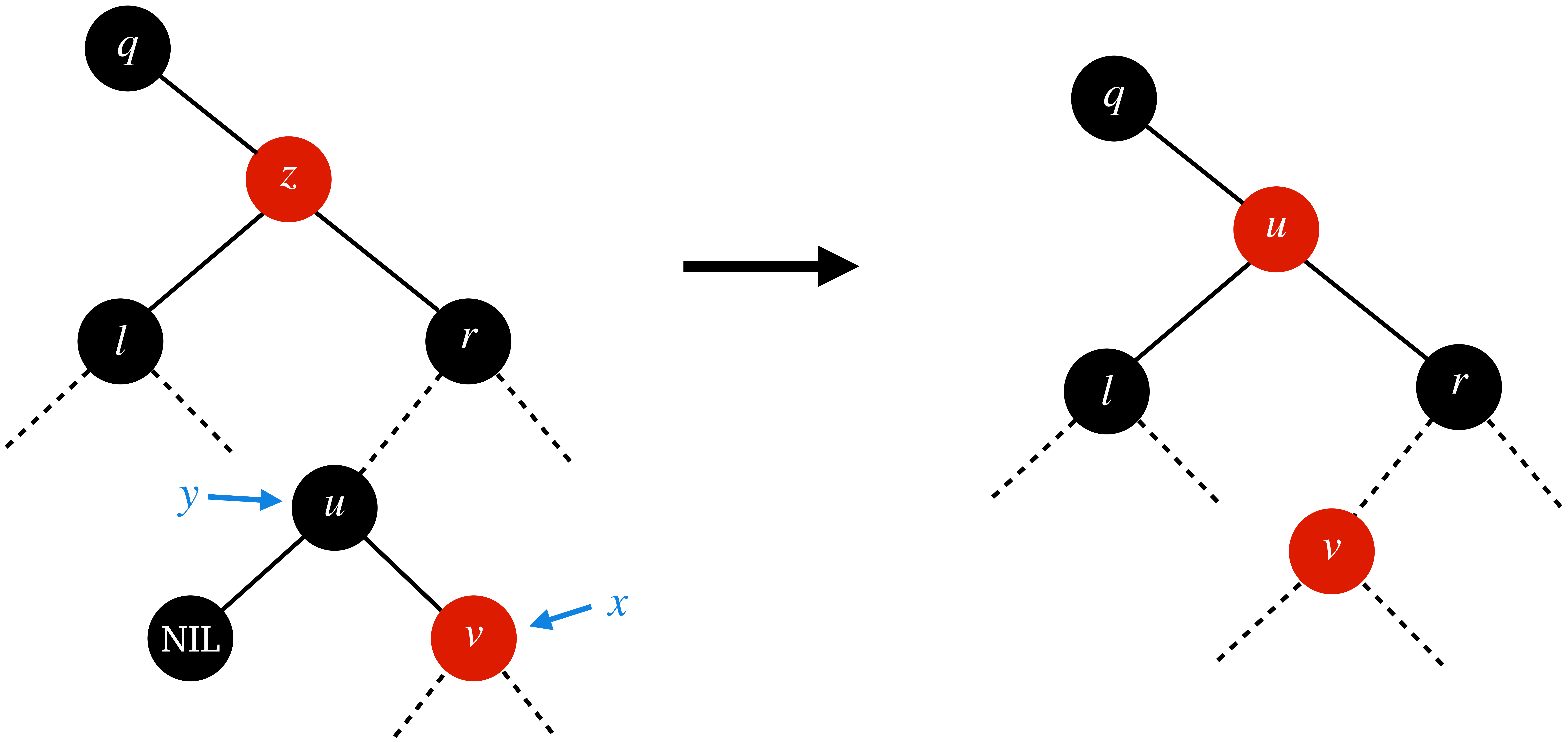

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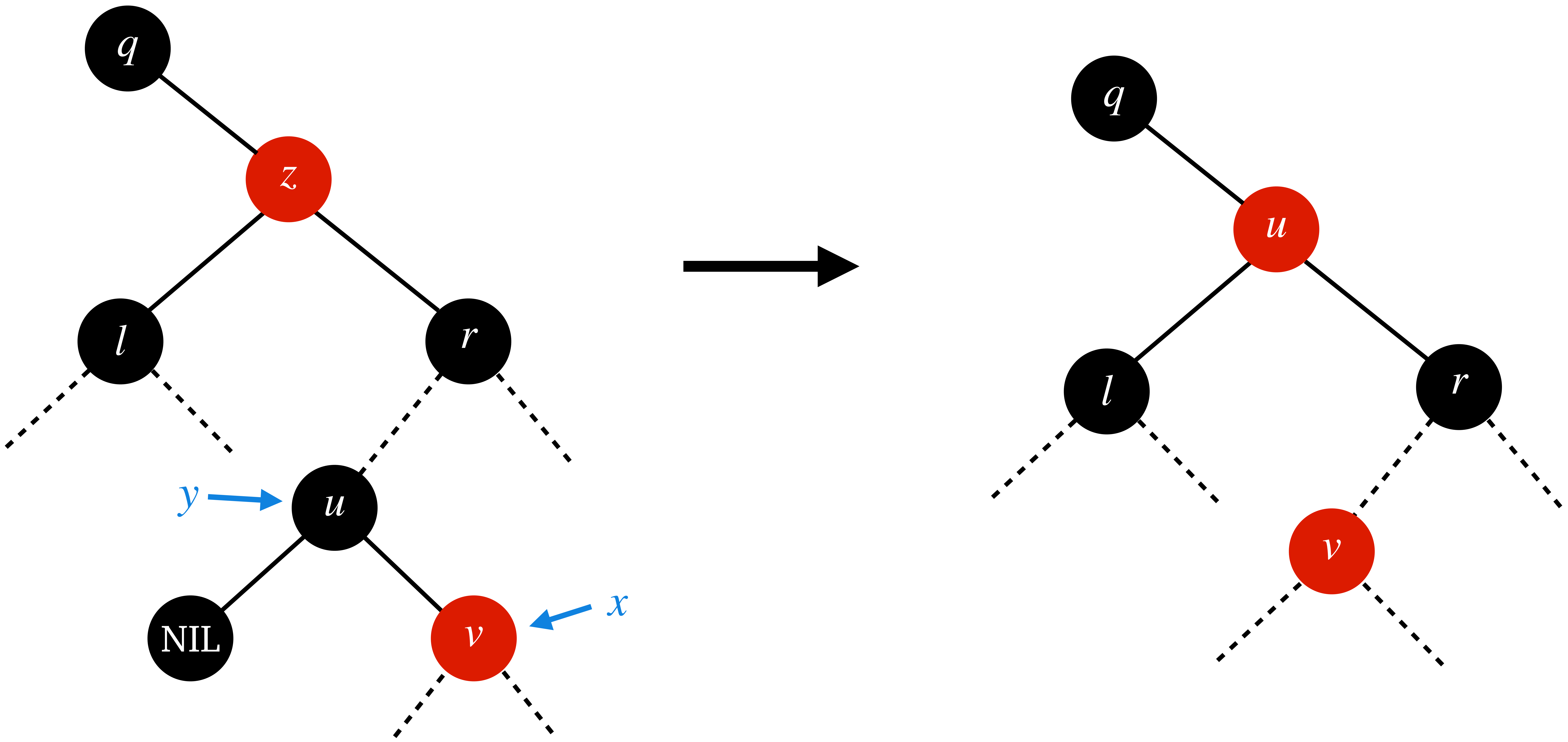

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RB-Trees: Deletion

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Skeleton for Deletion:

- Find y and x.
- If it's **Case 3**, replace z with y.

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Skeleton for Deletion:

- Find y and x.
- If it's **Case 3**, replace z with y.
- Remove y.

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Skeleton for Deletion:

- Find y and x.
- If it's **Case 3**, replace z with y.
- Remove y.
- Start fix ups from x depending on the colour of y.